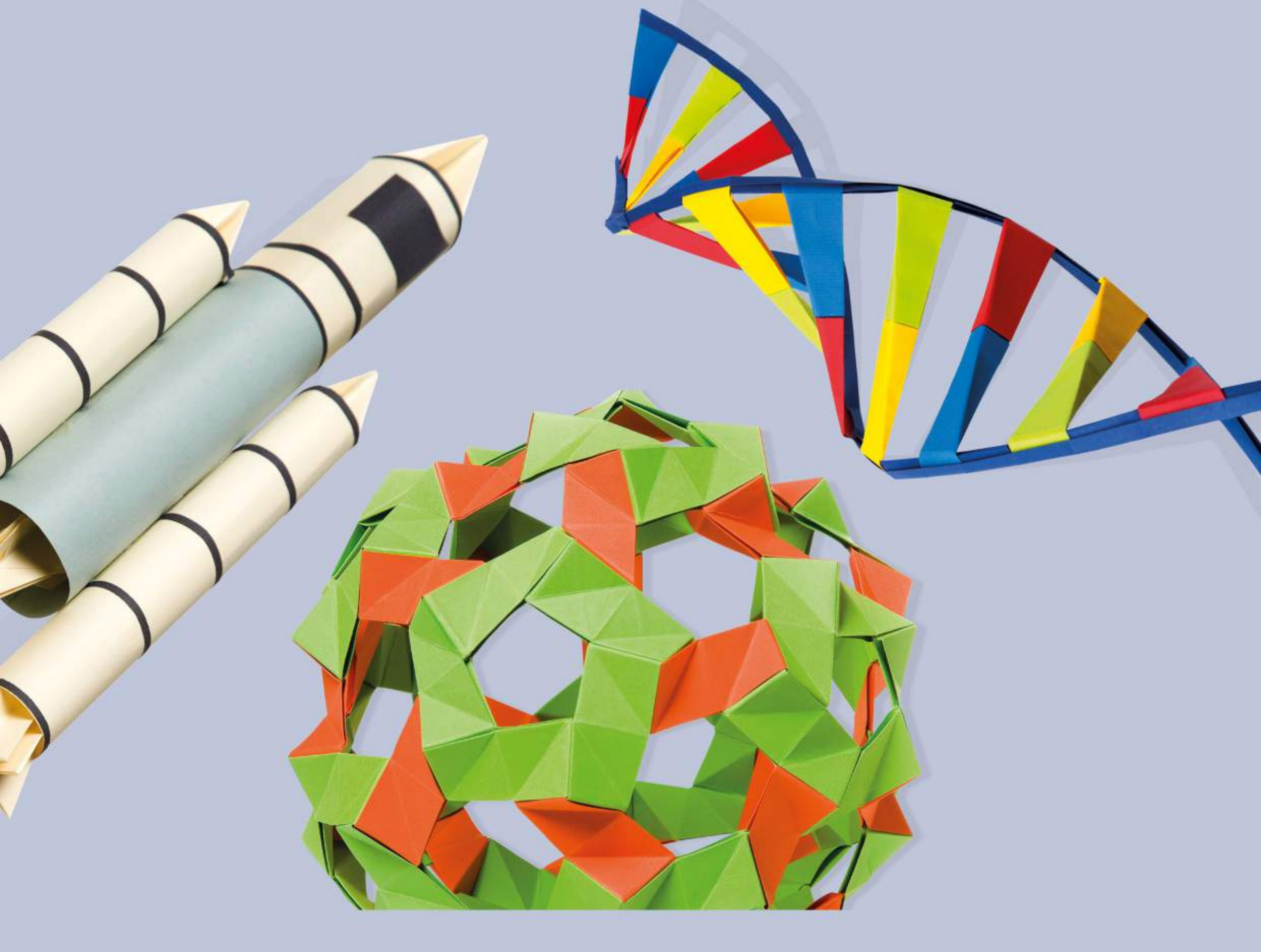
edexcel

Guide to Maths for Scientists



GCSE (9-1) Sciences

Pearson Edexcel Level 1/Level 2 GCSE (9-1) Sciences

Guide to Maths for Scientists

Contents

Introduction

This guide to maths for scientists outlines the content that students will have covered in their maths lessons throughout KS3 and KS4. You can use this guide to help you understand how different areas are approached in maths, and therefore support your teaching of mathematical content in science lessons.

The content is split into distinct mathematical concepts. Each chapter takes you through the terminology used in that area, as well as examples taken from Pearson maths textbooks to show you the methods students should be familiar with when solving mathematical problems.

1. Statistical graphs, charts and tables: data, bar charts, frequency tables and diagrams, pie charts, histograms

1.1 Data

Demand

All students learn the difference between discrete and continuous data in KS3. They also come across categorical data.

Terminology

- Data is either qualitative (descriptive) or quantitative (numerical) discrete or continuous data. **Inconsistency:** In science discrete data may be called discontinuous data.
- Discrete data can only take certain values, e.g. whole numbers, or shoe sizes. Continuous data is measured, e.g. mass, length, time, and can take any value.
- Categorical data is where there is no numerical value, but data can still be sorted into groups, e.g. eye colour.

1.2 Bar charts

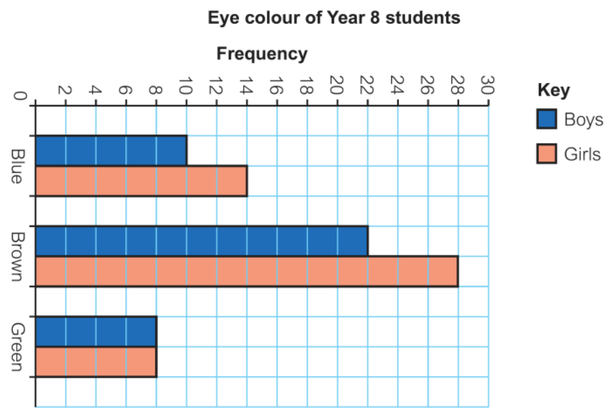
Demand

All students learn to draw and interpret bar charts for discrete and continuous data in KS3.

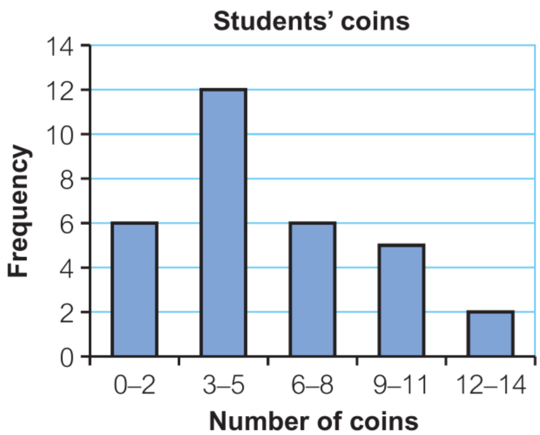
Lower ability maths students (KS3 and KS4) may need help with interpreting scales on axes given in e.g. thousands (i.e. 2.2 thousand = 2200) or millions.

Approach

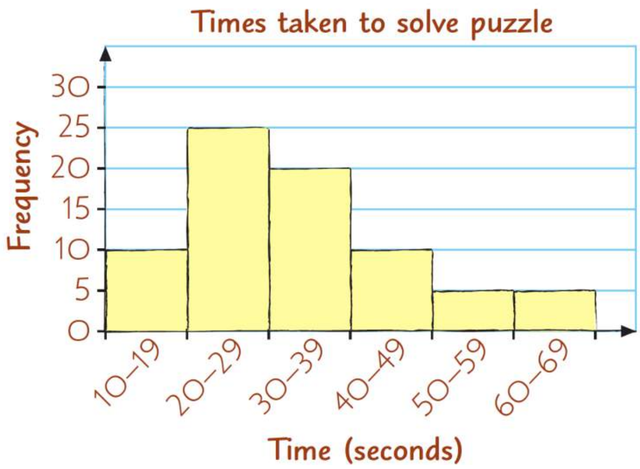
- Can show qualitative or quantitative discrete or continuous data.
- One axis is usually labelled 'Number of …' or Frequency.
- Frequency is usually shown on the vertical axis (but can be on the horizontal axis with the bars in the chart shown horizontally).
- Bars are of equal width.
- For discrete or qualitative data there are gaps between the bars.
- A bar-line graph, for discrete or qualitative data, uses lines instead of bars. It can be used to save time drawing the bars.
- For continuous data there are no gaps between the bars.
- In questions on interpreting proportions from bar charts, ask for the fraction or percentage of students with brown eyes, not 'the proportion of students'.



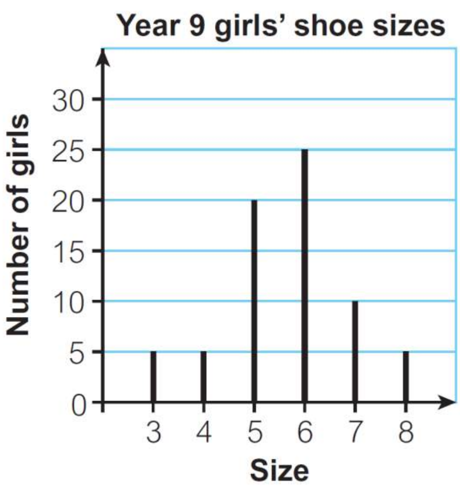
Horizontal bar chart for qualitative data



Bar chart for discrete data – gaps between the bars



Bar chart for continuous data – no gaps between the bars



Bar-line chart – for discrete data only

1.3 Frequency table

Demand

All students learn to draw and interpret frequency tables for discrete and continuous data in KS3.

Approach

- A table of data that shows the number of items, or frequency of each data value or each data group.
- Data can also be grouped. For discrete data use groups such as 0–5, 6–10, etc. For continuous data use groups such as $0 \le t < 10$, $10 \le t < 20$. The groups must not overlap.
- In maths, students learn that it is best to group numerical data into a max of 6 groups. If you need them to group data differently, tell them how many groups of equal width to group it into.

Shoe size	Frequency
3	3
4	5
5	7
6	10
7	10
8	6
9	1

Frequency table, ungrouped discrete data

Science mark	0–10	11–20	21–30	31–40	41–50
Frequency	4	13	17	19	7

Frequency table, grouped discrete data

Distance (*d* metres)	Frequency
$10 \le d < 20$	2
$20 \le d < 30$	6
$30 \le d < 40$	15
$40 \le d < 50$	20
$50 \le d < 60$	4

Frequency table, grouped continuous data.

1.4 Frequency diagram

Demand

All students learn to draw and interpret bar charts for discrete and continuous data in KS3.

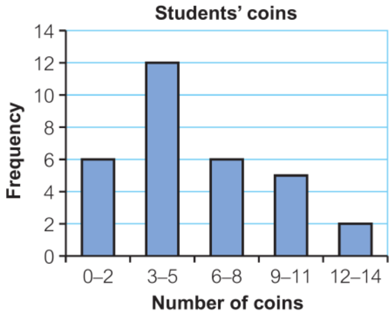
Lower ability maths students (KS3 and KS4) may need help with interpreting scales on axes given in e.g. thousands (i.e. 2.2 thousand = 2200) or millions.

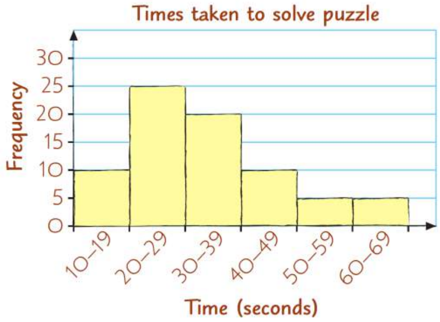
Inconsistency

In KS3 maths, the name 'Frequency diagram' is not used.

- Another name for a bar chart where the vertical axis is labelled Frequency.
- Can be used to show discrete or continuous data.

These two bar charts could also be called frequency diagrams:





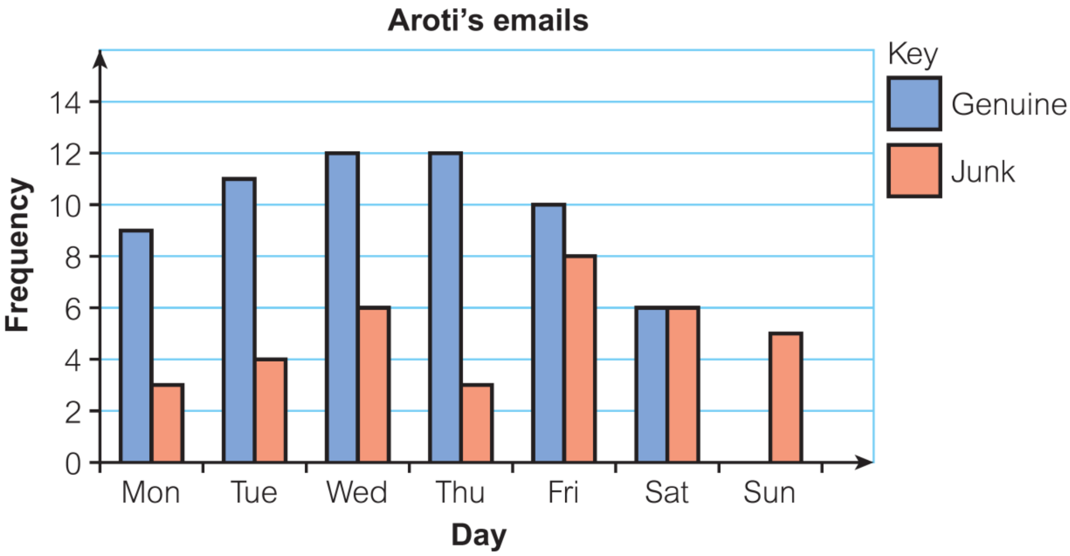
1.5 Comparative bar chart

Demand

All students learn to draw and interpret comparative bar charts in KS3.

Approach

- Compares two or more sets of data.
- Uses different coloured bars for each set of data.
- Needs a key to show what each colour bar represents.



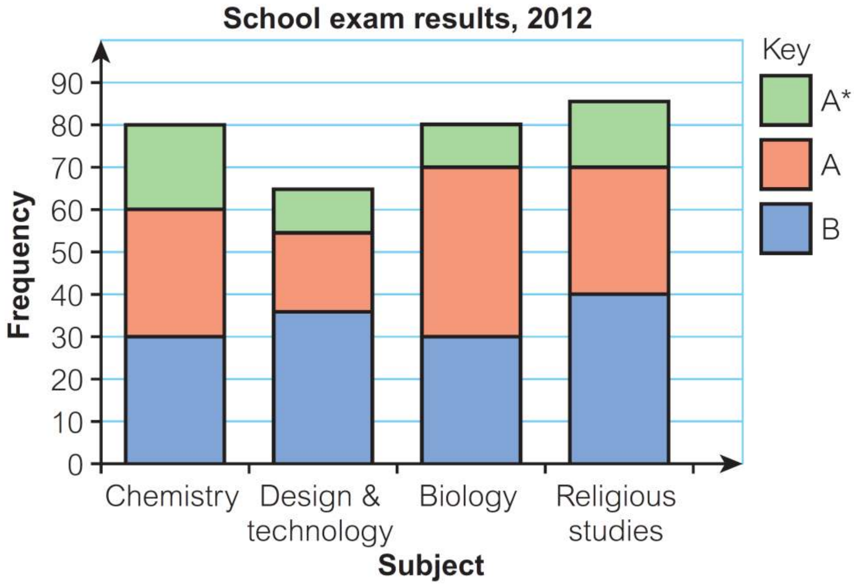
1.6 Compound bar chart

Demand

All students learn to draw and interpret compound bar charts in KS3.

Approach

- Combines different sets of data in one bar.
- Needs a key to show what each colour section represents.
- In questions on interpreting proportions in compound bar charts, ask for the fraction or percentage of chemistry students getting A*, not 'the proportion of students'.



1.7 Histogram

Demand

In maths, students do not meet histograms until KS4, although the bar charts they draw in KS3 for grouped continuous data could also be called histograms.

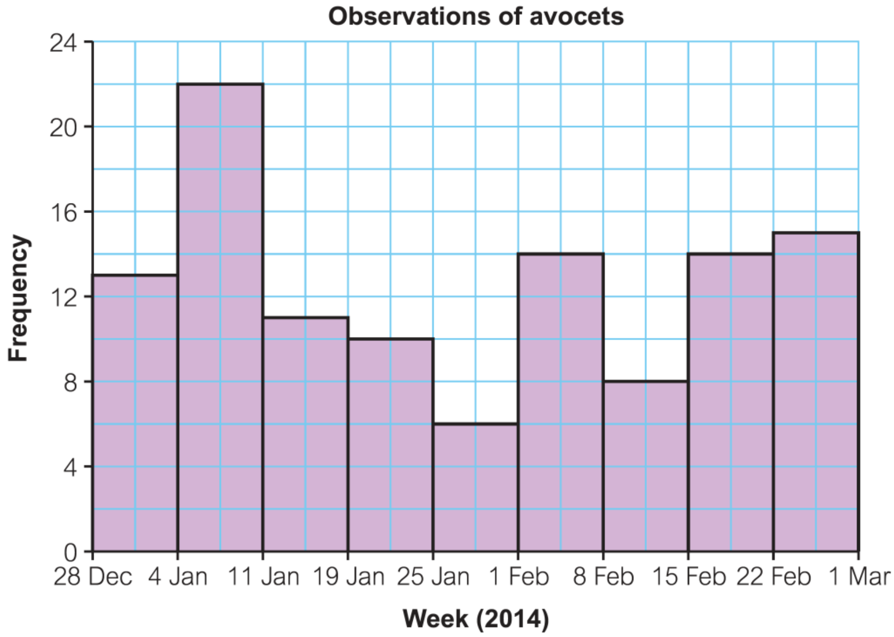
Histograms with unequal width bars/groups, where frequency density is plotted on the vertical axis, are only covered in Higher GCSE Maths, not Foundation.

Approach

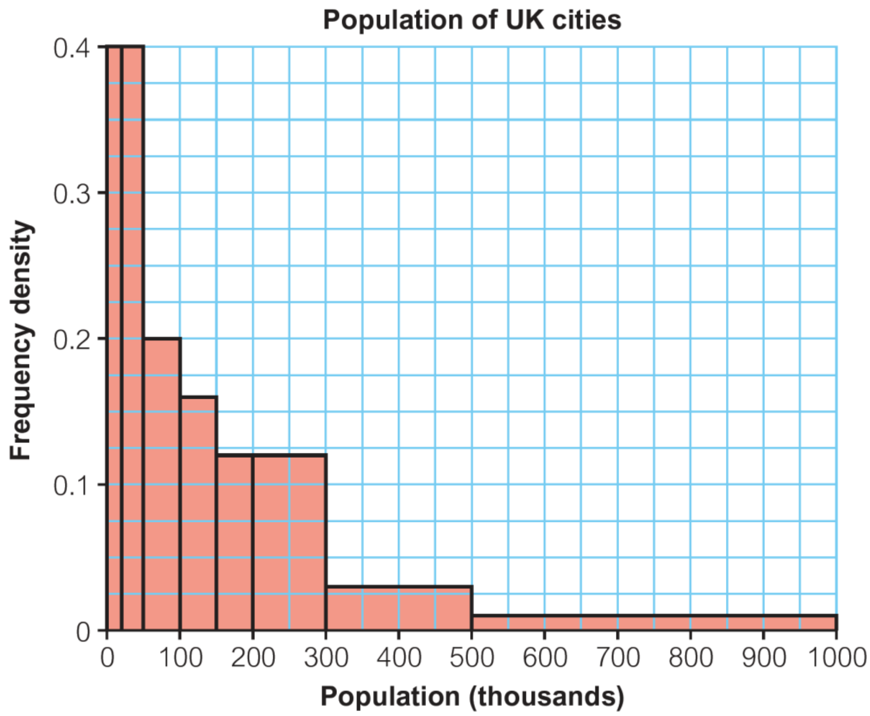
- Can be drawn for grouped continuous data where groups/bars are of equal width.
- No gaps between the bars.
- If groups/bars are of unequal width, the vertical axis is labelled frequency density, which is calculated as

$$\frac{\text{number in group}}{\text{group width}}$$

  and the area of the bar is proportional to the number of items it represents (frequency).



Histogram with equal width bars/groups



Histogram with unequal width bars/groups – used when the data is grouped into classes of unequal width

Drawing a histogram

From Pearson GCSE Mathematics Higher:

Key point 12

In a **histogram** the area of the bar represents the frequency. The height of each bar is the frequency density.

$$\text{Frequency density} = \frac{\text{frequency}}{\text{class width}}$$

Example 4

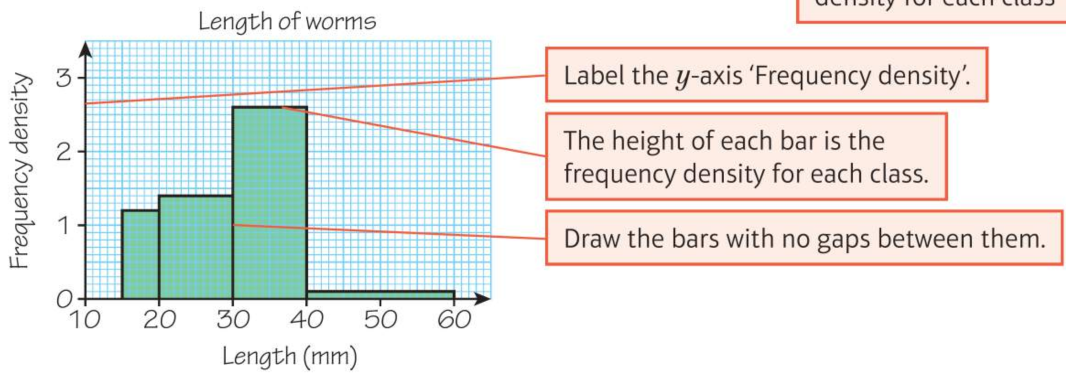
The lengths of 48 worms are recorded in this table.

Length, x (mm)	$15 < x \leqslant 20$	$20 < x \leqslant 30$	$30 < x \leqslant 40$	$40 < x \leqslant 60$
Frequency	6	14	26	2

Draw a histogram to display this data.

$6 \div 5 = 1.2,\ 14 \div 10 = 1.4,\ 26 \div 10 = 2.6,\ 2 \div 20 = 0.1$

Work out the frequency density for each class



1.8 Pie charts

Demand

In KS3 all students should learn how to construct a simple pie chart.

Lower ability students will probably struggle with working out the angles as they will not have learned how to calculate percentages or fractions that are not nice round numbers.

All students draw and interpret pie charts in GCSE Maths.

Approach

Drawing a pie chart

Taken from Pearson GCSE Maths Foundation:

Example 5

The table shows the match results of a football team. Draw a pie chart to represent the data.

Result	Won	Drawn	Lost
Frequency	28	12	20

Total number of games = 28 + 12 + 20 = 60

$$\div 60 \left(\begin{array}{c} 60 \text{ games} : 360° \\ 1 \text{ game} : 6° \end{array} \right) \div 60$$

The total number of games is the total frequency.

1 game = 360 ÷ 60 = 6°

Work out the angle for one game.

Won: 28 × 6° = 168°

Drawn: 12 × 6° = 72°

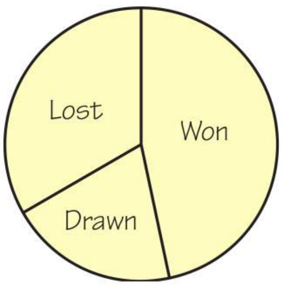
Lost: 20 × 6° = 120°

Work out the angle for each result.

Check: 168 + 72 + 120 = 360

Check that your angles total 360°.

Team results

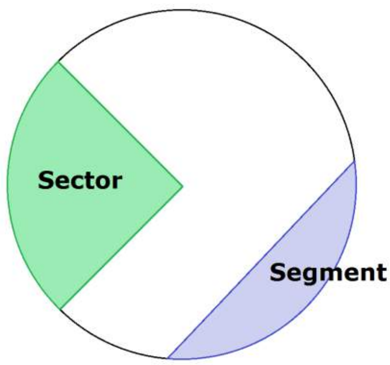


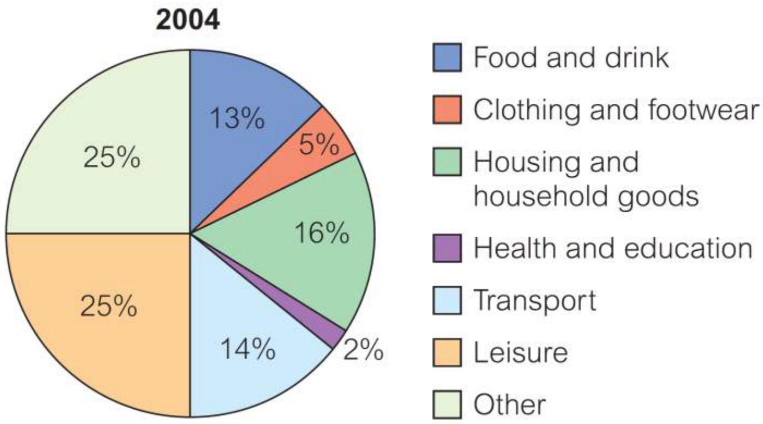
Draw the pie chart. Give it a title and label each section, or make a key.

In questions asking students to interpret a pie chart, ask for the fraction or percentage who learn French, not 'the proportion' who learn French.

Terminology

- A pie chart is a circle divided into **sectors**. NB – a 'slice' of a pie is a sector, not a segment.
- The angle of each sector is proportional to the number of items in that category
- Shows proportions of a set of data, e.g. fraction or percentage of waste recycled
- May need a key.
- In questions asking students to interpret a pie chart, ask for the fraction or percentage who learn French, not 'the proportion' who learn French.





Pie chart with a key

1.9 Drawing graphs and charts to display data

Demand

Choosing a suitable graph or chart to draw to display data is above level 6 in maths.

Lower ability students in KS3 and Foundation GCSE maths students may need some guidance on the type of chart to draw for given sets of data.

Lower ability maths students may also need help with choosing suitable scales for axes.

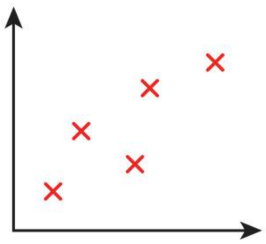
2. Graphs including lines of best fit, proportionality, gradients, relationships and correlations

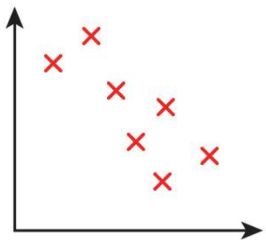
2.1 Scatter graphs

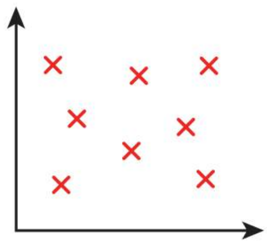
Terminology and approach

Inconsistency: In maths we usually call these scatter graphs, not scatter diagrams.

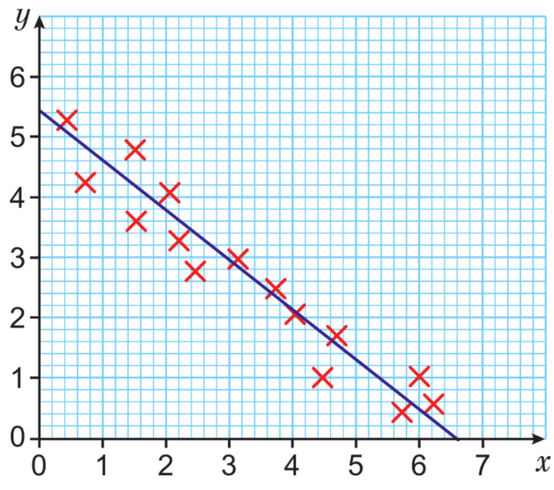
- A scatter graph plots two sets of data on the same graph to see if there is a relationship or correlation between them.
- Scatter graphs can show positive, negative or no correlation.


Positive correlation

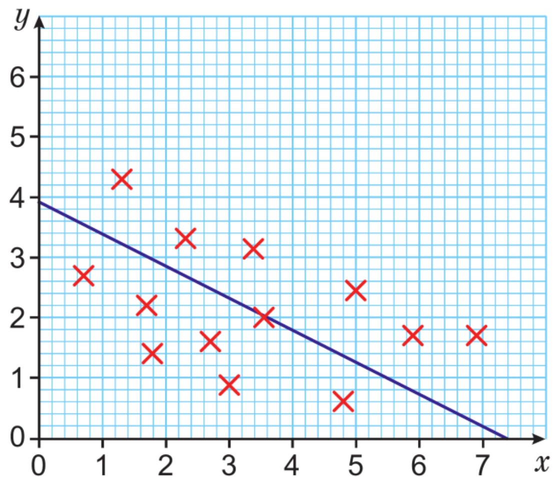

Negative correlation


No correlation

- Correlation is when two sets of data are linked. For example, when the values increases as the other increases, or when one value decreases as the other increases.
- In maths, points on scatter graphs are plotted with crosses.
- The line of best fit follows the shape of the data and has roughly the same number of crosses above and below the line. There may also be crosses on the line.



Strong correlation

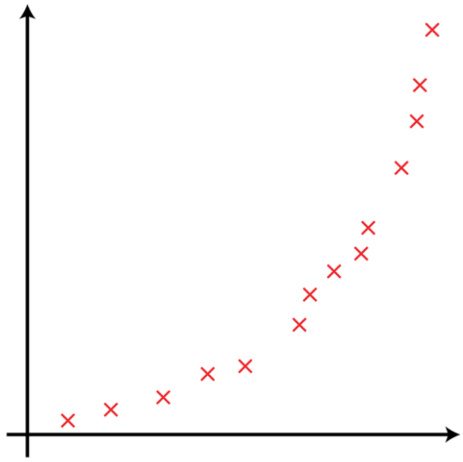


Weak correlation

- **Inconsistency:** In maths, when interpreting a scatter graph an acceptable answer is 'shows positive correlation', unless the question explicitly asks for this to be explained in context.
- The line of best fit shows a relationship between two sets of data.
- When the points on a scatter graph are on or close to a straight line:
  - o There is strong correlation between the variables.

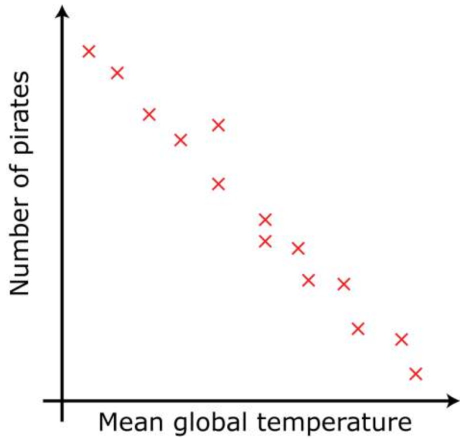
- o There could be a linear relationship between the variables, e.g. $y = mx$ or $y = mx + c$. The equation of the line of best fit describes this relationship.

- When the points on a scatter graph are not close to a straight line there may be another relationship between the variables.

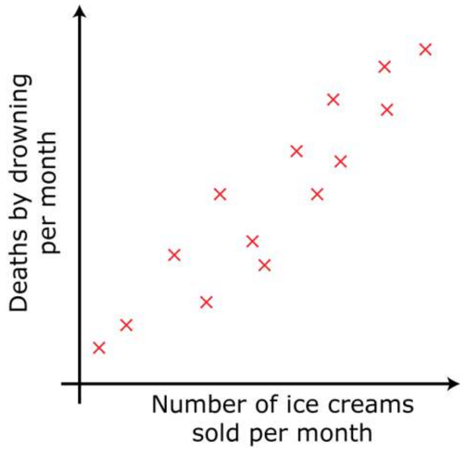


Scatter graph showing a possible non-linear relationship

- Correlation does not imply causation. Sometimes there may be another factor that affects both variables, or there may be no connection between them at all.



There is a negative correlation between number of pirates and mean global temperature, but it is unlikely that one causes the other.



There is positive correlation between number of ice-creams sold and death by drowning, but it is unlikely that one causes the other. A more likely explanation is a third factor – temperature. On hot days more people buy ice creams and more people swim, leading to increased numbers of drownings.

Approach

Drawing scatter graphs

Lower ability maths students would not be expected to know which variables to put on which axis for a scatter graph. They may need help with deciding which is the independent variable, and reminder that this goes on the horizontal axis.

Drawing a line of best fit

Place your ruler on the graph, on its edge. Move the ruler until it is following the shape of the data, with roughly the same number of points above or below it. Ignore any points on the line.

Common error

Students often try to make their lines of best fit go through (0, 0). A line of best fit does not necessarily pass through the origin.

2.2 Interpreting scatter graphs

Demand

All students should learn to interpret scatter graphs in KS3.

Lower ability students at GCSE would not be expected to know which variables to put on which axis for a scatter graph.

Students are not expected to find equation of a curve of best fit in GCSE Maths.

At GCSE Higher level they would be expected to state that there is a possible non-linear relationship if the points on a scatter graph closely follow a smooth curve.

They will have learned about correlation and causation in KS3 maths.

Common error

Students often find interpreting scatter graphs difficult as they do not know how to put into words what the graph shows, so it is good to give them examples of this, or at least sentences to copy and complete, such as:

the higher the temperature the _________ the ______ the speed.

You can also use statements such as:

the taller the..., the longer the ....

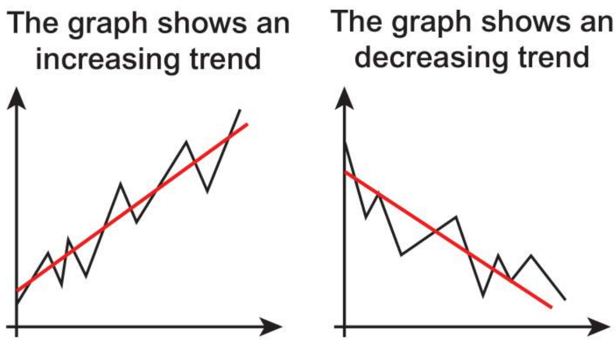
2.3 Line graphs

Demand

Lower ability students at KS3 would not be expected to know which variables to put on which axis for a scatter graph. They may need help with deciding which is the independent variable, and reminder that this goes on the horizontal axis.

Terminology

- In maths, a line graph that shows how a variable changes over time (i.e. with time on the horizontal axis) is often called a time-series graph.
- Line graphs can show trends in data. The trend is the general direction of change, ignoring individual ups and downs.



2.3.1 Drawing line graphs

Demand

Lower ability students at KS3 would not be expected to know which variables to put on which axis for a line graph. They may need help with deciding which is the independent variable, and reminder that this goes on the horizontal axis.

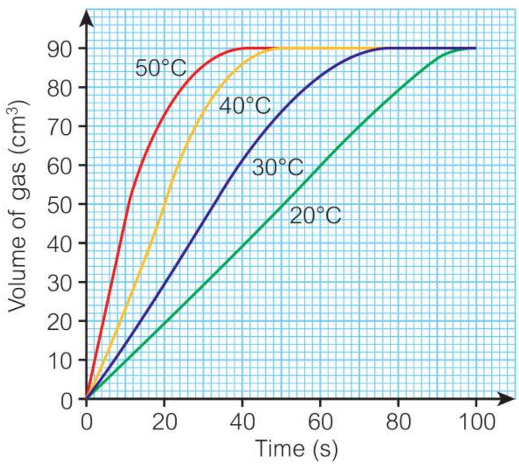
Choosing a suitable graph or chart to draw to display data is above level 6 in maths.

Lower ability students in KS3 and Foundation GCSE maths students may need some guidance on the type of chart to draw for given sets of data.

Lower ability maths students may also need help with choosing suitable scales for axes.

Approach

- In maths, students draw graphs on squared or graph paper.
- They plot points with crosses (×).
- They join points with straight lines or a smooth curve – the question needs to tell them which.
- All graphs should have labels on the axes and a title.
- When more than one data set is shown, the lines could be e.g. one solid and one dashed. The graph will need a key to explain solid/dashes. Alternatively, label each line e.g. Set 1 and Set 2.



Line graph showing more than one data set.

2.3.2 Interpreting line graphs

Demand

All students interpret line graphs in KS3.

Lower ability maths students (KS3 and KS4) may need help with interpreting scales on axes given in e.g. thousands (i.e. 2.2 thousand = 2200) or millions.

At KS4 all students will have limited experience of interpreting real life graphs that dip below zero.

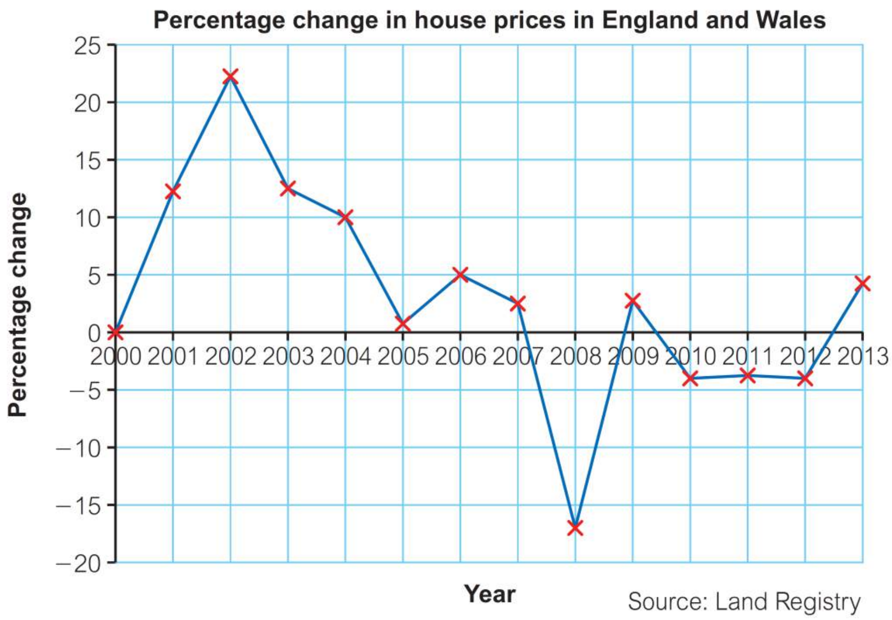
Only higher ability maths students are likely to have seen graphs with two different vertical scales to read from.

Approach

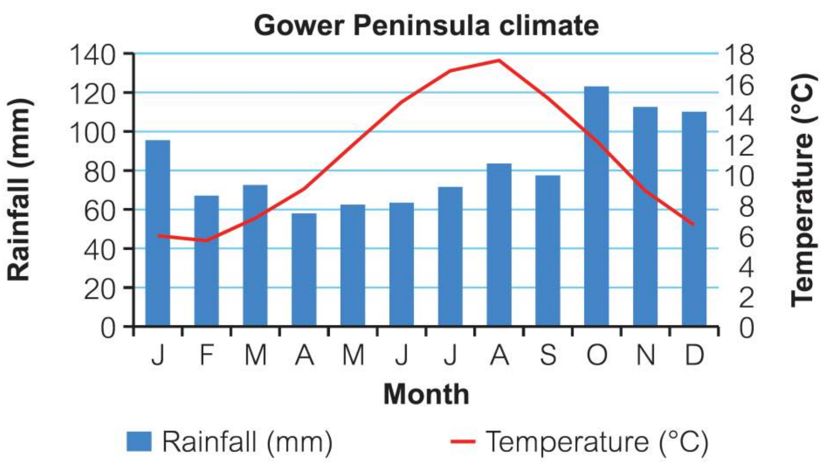
Students interpret 'real life' graphs in maths, in a variety of contexts.

They may be less familiar with:

- Graphs showing negative values, e.g.



- Two types of graph on one set of axes, and two different vertical axes for the same graph:



- Ensure the terms used in the question match the labels on the graph.

2.4 Recognising proportional relationships from graphs

Demand

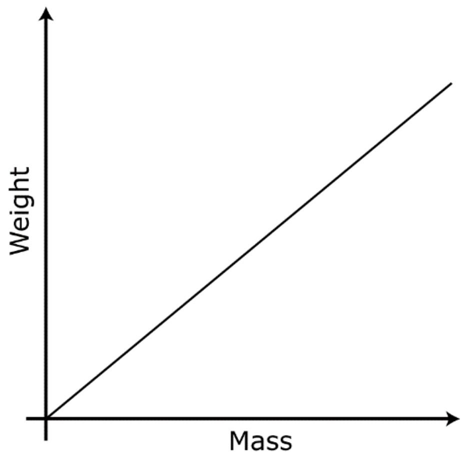
In year 9, all but bottom set maths students should know that a straight line graph shows the two variables are in direct proportion (please use direct proportion, not just proportion). Only top sets maths students will have met graphs showing inverse proportion.

In KS4 all students should learn that the origin is the point (0, 0).

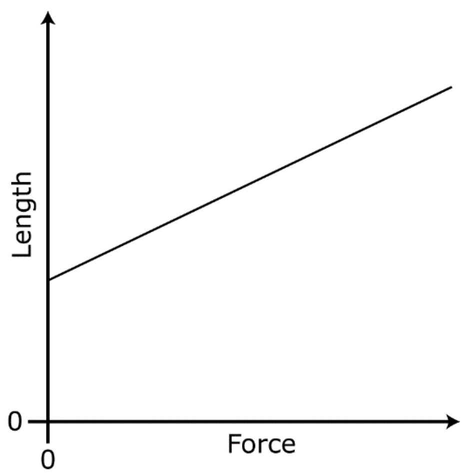
In KS4 all students will meet graphs showing inverse proportion.

Terminology

- A straight line graph through the origin (0, 0) shows that the two variables are in **direct proportion**.
  When one doubles, so does the other. When one halves, so does the other.
  The relationship is of the form $y = mx$, where m is the gradient of the graph.
- A straight line graph not through the origin shows a **linear relationship**.
  The relationship is of the form $y = mx + c$, where m is the gradient of the graph and c is the y-intercept (where the graph crosses the y-axis).



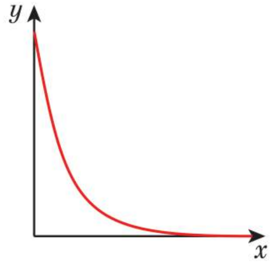
Graph showing a directly proportional relationship between weight and mass.
The gradient is $g \approx 10$.



Graph showing a linear relationship – how the length of a spring changes as the force applied increases.
The y-intercept is the initial length of the spring.
The gradient is the increase in length for unit increase in force.

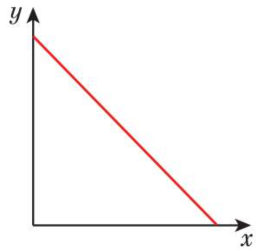
- A graph of the form $y = \frac{1}{x}$ or $y = \frac{k}{x}$ shows that the two variables are in inverse proportion.

- **<u>Inconsistency</u>:** in maths we do not say 'indirect proportion'.
  When one doubles, the other halves. When one halves, the other doubles.
  The relationship is of the form $y = \frac{k}{x}$, where k is a constant.



Graph showing an inversely proportional relationship

- **<u>Common error</u>**: A straight line graph with negative gradient shows a linear relationship – not inverse proportion. This is a fairly common misconception.



2.5 Rate of change graphs – including distance-time and velocity (speed)-time graphs

Terminology

Words to describe graph lines:
vertical, horizontal (not level), gradient, steep, steeper, less steep (not shallow). You can describe a graph line as 'sloping' but do not use 'slope' when you mean 'gradient' – i.e. a numerical value.

2.5.1 Distance-time graphs

Demand

All students meet distance-time graphs in KS3.

By the end of KS3 they should know that the steeper the gradient, the faster the distance is changing, i.e. the faster the speed and that a horizontal line means the distance from the start is not changing, so the object is stationary.

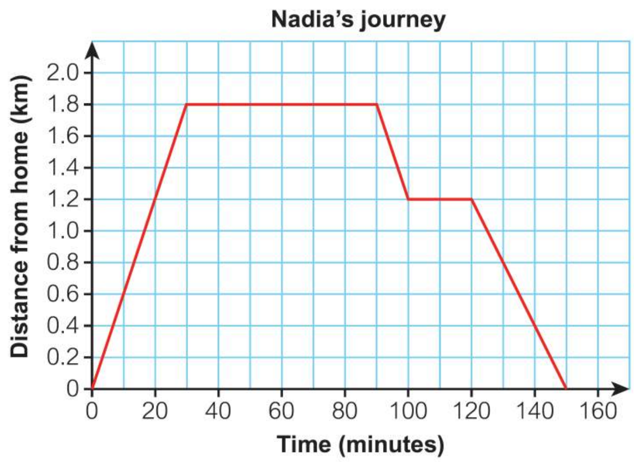
Higher and middle ability maths students learn how to calculate speed from a distance time graph in KS3. Lower ability students do not.

Foundation GCSE Maths students learn to calculate speed from a distance-time graph in KS4.

Approach

Calculating speed from a simple distance-time graph

Using arrow diagrams



For first part of journey:

×2 1.8 km in 30 mins ×2

so 3.6 km in 1 hour

Speed = 3.6 km/h

For first part of return journey:

×6 0.6 km in 10 mins ×6

so 3.6 km in 60 mins

Calculating speed from more complex distance-time graphs

On a distance-time graph, speed = gradient.

See below for calculating the gradient of a graph.

Calculating average speed from a distance-time graph

Read values from the graph and use the formula

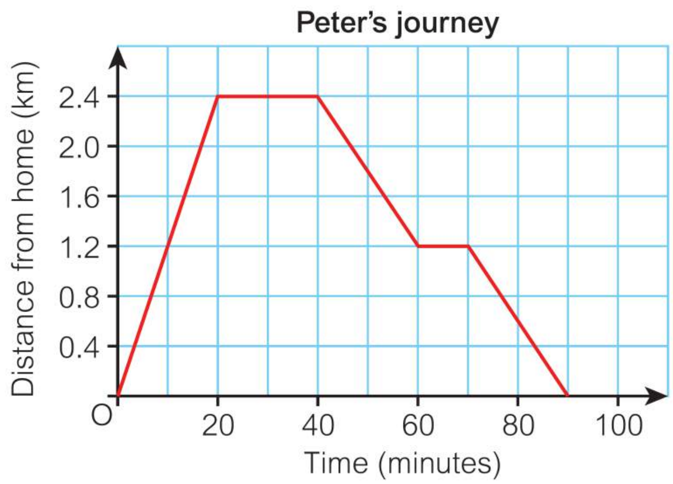
Average speed = $\frac{\text{total distance}}{\text{total time}}$

Assumptions made in drawing distance-time graphs

That speed is constant.

Terminology

- The vertical axis shows the distance from the starting point.
- The horizontal axis shows the time taken.
- The gradient is the speed. Use phrases such as: the steeper the line, the faster the speed.
- A horizontal line shows the object is stationary (its distance from the starting point is not changing).
- A line sloping upwards shows the object is moving away from the starting point.
  A line sloping downwards shows the object is moving towards the starting point.



2.5.2 Speed-time graphs

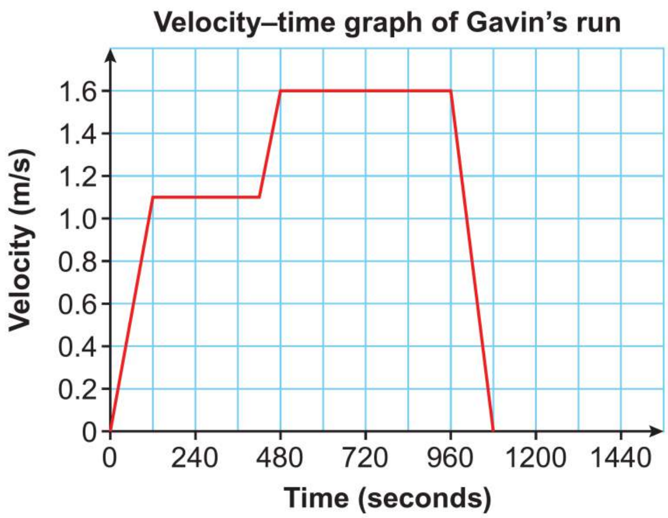
Inconsistency: in maths these are usually called velocity–time graphs. Velocity is defined as speed in a given direction.

Demand

Velocity-time graphs are new at GCSE from 2015.

All students should meet velocity–time graphs in KS4.

Approach



Calculating acceleration and deceleration from a velocity–time graph

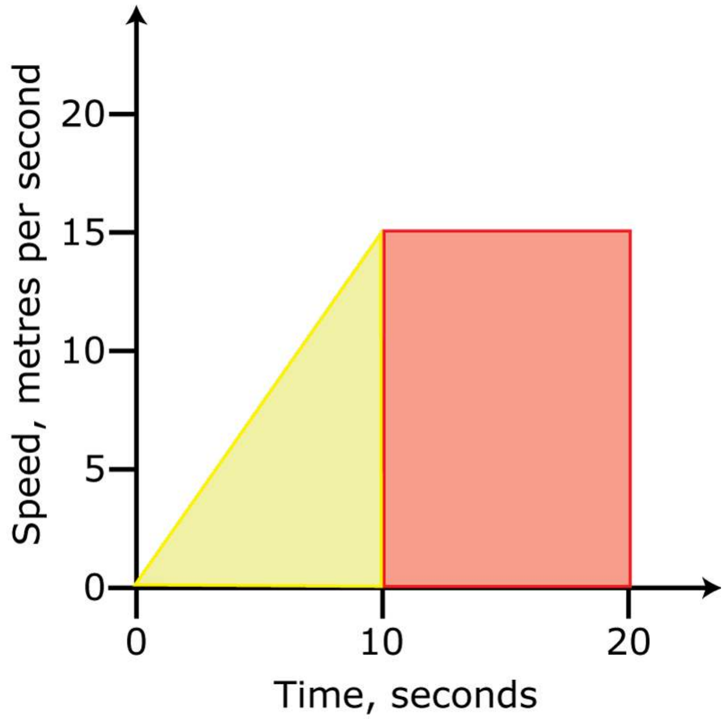
On a speed-time graph, acceleration = gradient.

See below for calculating the gradient of a graph.

Negative acceleration is deceleration.

Calculating distance travelled from a velocity–time graph

In a velocity-time graph, the distance travelled is the area under the graph.



Between 10 and 20 seconds:

distance travelled = area of rectangle = 15 m/s × 10 s = 150 m

For the first 10 seconds:

distance travelled = area of triangle = $\frac{1}{2}$ × base ×$\frac{1}{2}$ height

= $\frac{1}{2}$ × 10 s × 15 m/s

= 75 m

Assumptions made in drawing velocity-time graphs

That velocity and acceleration are constant.

Terminology

- The vertical axis shows the velocity.
- The horizontal axis shows the time taken
- The gradient is the acceleration. The steeper the line, the greater the acceleration.
- A horizontal line shows the object is travelling at constant velocity (its velocity is not changing).
- A line sloping upwards shows the object is accelerating.
- A line sloping downwards shows the object is decelerating.
- The area under a speed-time graph is the distance travelled. **Inconsistency:** In maths we say the area under a velocity-time graph is the displacement.

2.6 Calculating the gradient of a straight-line graph

Demand

All students learn to find the gradient of a line in KS3.

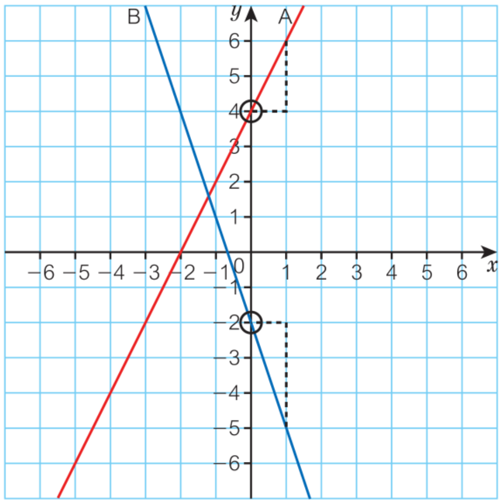
Approach

Calculating gradient of a simple graph

The gradient of a line is how far you go up for every 1 you go across.

For line A: for every 1 you go across, you go 2 up. Gradient = 2.

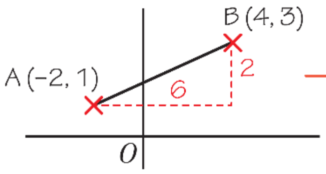
For line B: for every 1 you go across, you go 3 down. Gradient = −3.



More complex gradient and *y*-intercept

Example 3

Find the gradient of the line joining the points A (−2, 1) and B (4, 3).



Sketch a diagram. Draw in lines across and up. Work out the distances across and up.

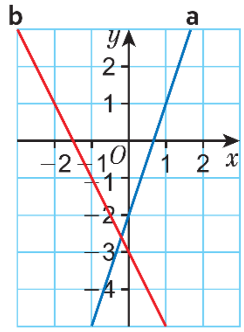
$$\text{Gradient} = \frac{\text{total distance up}}{\text{total distance across}} = \frac{2}{6} = \frac{1}{3}$$

Key point 5

A **linear equation** produces a straight line graph. The equation of a straight line is $y = mx + c$, where m is the gradient and c is the y-intercept.

Example 4

Write the equation of each line.



a $y = mx + c$

$m = 3$ — Work out the gradient, m.

$c = -2$ — The line crosses the y-axis at −2.

Equation of line is $y = 3x - 2$ — Substitute $m = 3$ and $c = -2$ into $y = mx + c$

b $y = mx + c$

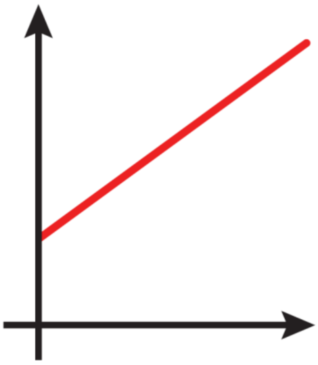
$m = -2$ — This line slopes down so its gradient is negative.

$c = -3$

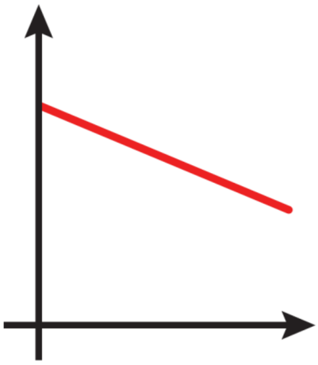
Equation of line is $y = -2x - 3$

Terminology

- Gradient is a measure of steepness of a line.
- In a rate of change graph, the steeper the gradient, the faster the rate of change.
- Positive gradients slope uphill from left to right.
- Negative gradients slope downhill from right to left.



positive gradient



negative gradient

2.7 Equation of a straight line

Demand

All students learn to find the equation of a straight line $y = mx + c$ at KS3.

Approach

- Equation of a straight line graph is $y = mx + c$, where m is the gradient and c is the y-intercept (where the line crosses the y-axis).

Finding the equation of a linear graph

Worked example

Write the equation of

a line A

b line B.

a $y = mx + c$

gradient, $m = 2$

y-intercept is (0, 4), so $c = 4$.

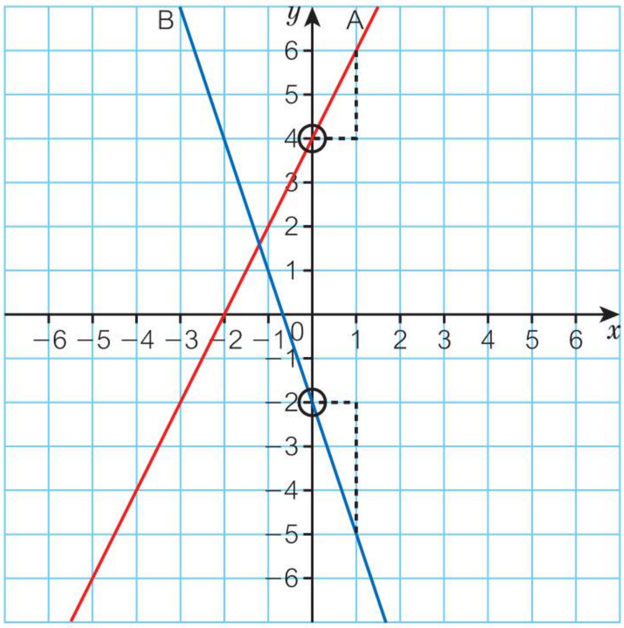
Equation of line A is $y = 2x + 4$.

b $y = mx + c$

gradient, $m = -3$

y-intercept is (0, −2), so $c = -2$.

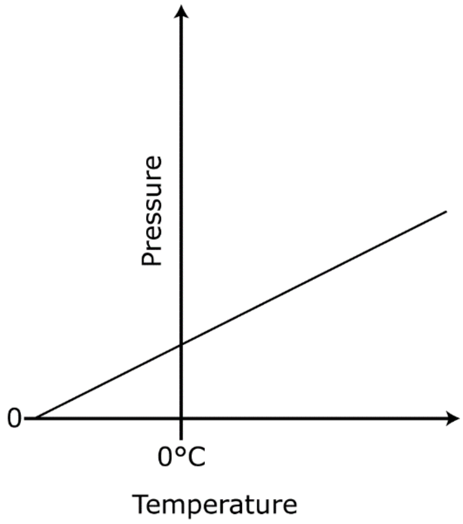
Equation of line B is $y = -3x - 2$.



Interpreting the equation of the line in the context of the graph

Example

The graph shows that there is a linear relationship between the pressure inside a container of gas and its temperature.



For this graph:

y is the pressure

m is the gradient of the line, so it tells you how much the pressure increases for each 1°C increase in temperature

$$y = mx + c$$

x is the temperature

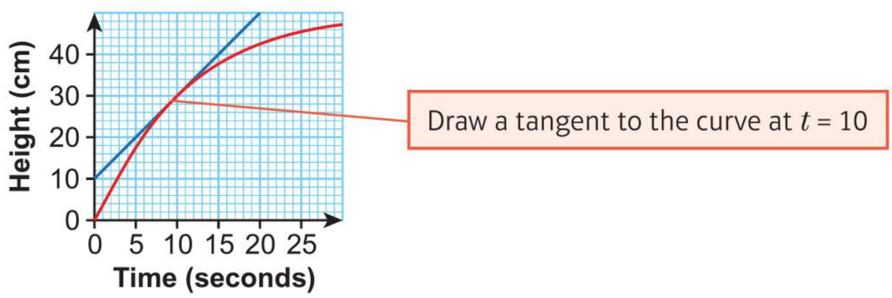
c is point at which the line crosses the vertical axis, so it tells you what the pressure is at 0°C

2.8 Calculating the gradient at a point on a curve

Demand

In maths, estimating the gradient for a curve or calculating the gradient at a point on a curve is in the new GCSE (hasn't been in previously). Only Higher GCSE Maths students will learn this, and not until the last chapter in our GCSE book.

Approach



$$\text{Gradient} = \frac{\text{change in } h}{\text{change in } t} = \frac{50 - 10}{20 - 0} = \frac{40}{20} = 2$$

Calculate the gradient of the tangent.

Understand the physical significance of area between a curve and the *x*-axis and measure it by counting squares as appropriate

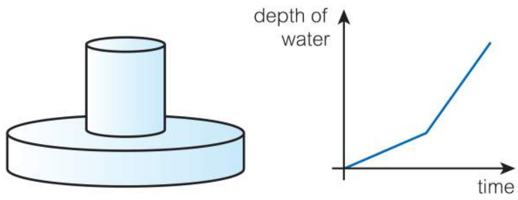
- In maths, area under a curve is in the new GCSE (hasn't been in previously). Only Higher GCSE Maths students will learn this, and not until the last chapter in our GCSE book.
- In maths they will estimate the area under a curve by dividing it into a number of trapezia, not by counting squares.
- They will interpret the area under a velocity-time curve as the distance travelled, but will not understand the physical significance of this area in other contexts.

2.9 Rate of change graphs

Demand

In KS3, top and middle sets maths students (not lower ability) will have learned to interpret rates of change graphs by considering the steepness of the graph, and when the change is faster/slower in examples like this:

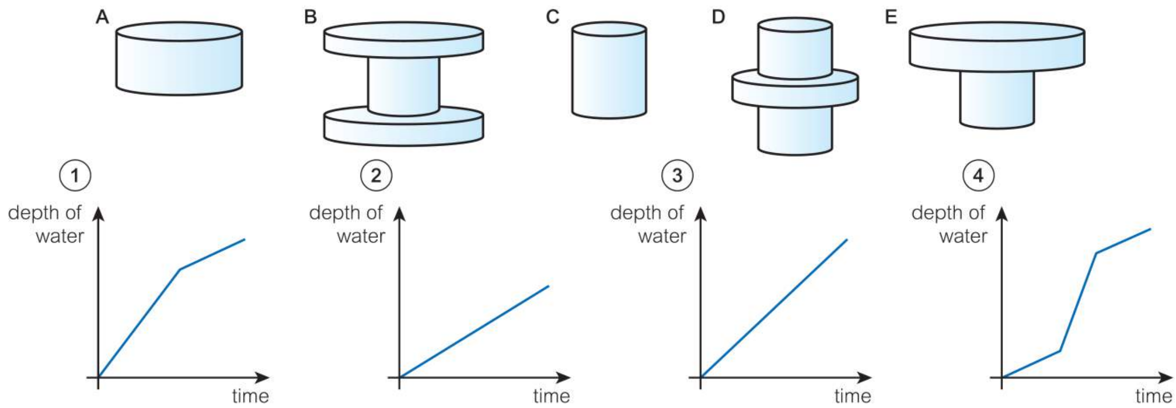
The graph shows how the depth of water in this container changes over time, when water is poured in at a steady rate.



a Which bit of the container fills fastest, the wide part or the narrow part?

b How can you tell when it is filling fastest from the graph?

c Match each of the graphs to the correct container.



d Which container has not been matched to a graph? Sketch a depth–time graph for this container.

At KS4 they learn to interpret the gradient of a line as the rate of change.

In maths, estimating the gradient for a curve or calculating the gradient at a point on a curve is in the new GCSE (hasn't been in previously).

3. Fractions, percentages, ratios

3.1 Fractions

Demand

All students learn how to add, subtract, multiply and divide decimals and find a fraction of a quantity in KS3.

They also learn how to convert fractions to decimals and vice versa, and use and interpret recurring decimal notation.

Lower ability students will not learn to find the reciprocal of a fraction at KS3. All students will learn this in GCSE Maths.

Approach

Convert a fraction to a decimal

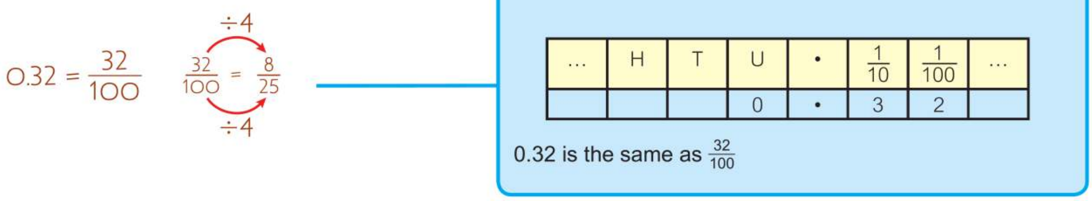
Divide the top number by the bottom number.

For example $\frac{3}{8} = 0.375$, $\frac{12}{50} = 0.24$

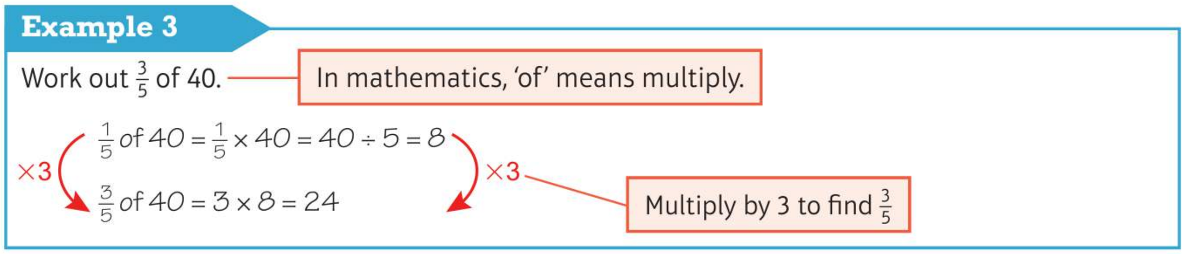
Convert a decimal to a fraction

Worked example

Write 0.32 as a fraction in its simplest form.



Calculate a fraction of a quantity



Reciprocals

The reciprocal of a number is 1 ÷ the number.

The reciprocal of 4 is $\frac{1}{4}$ or 0.25.

The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

The reciprocal of $\frac{1}{2}$ is $\frac{2}{1} = 2$.

Terminology

- Write fractions on two lines, i.e. $\frac{1}{1000}$ not 1/1000 on one line.
- Avoid 'cancelling' – instead, write fractions in their simplest form.
- In a fraction, the horizontal line means 'divide'. So $\frac{3}{5}$ means 3 ÷ 5. Understanding this helps students remember how to convert fractions to decimals.
- A 'dot' over a decimal value shows the number recurs, e.g. $0.\dot{6}$ means 0.66666…
- A dot over two decimal values shows the numbers between the dots recur, e.g. $0.\dot{1}\dot{5}$ means 0.151515… and $0.2\dot{4}75\dot{1}$ means 0.247514751…
- The reciprocal of a number is 1 ÷ the number. For fractions, this means the reciprocal of $\frac{a}{b} = 1 \div \frac{a}{b} = 1 \times \frac{b}{a} = \frac{b}{a}$.
- Dividing by a fraction is the same as multiplying by its reciprocal.

3.2 Percentages

Demand

In KS3, students in lower maths sets will only have written one number of a percentage of another where the larger number is a multiple or factor of 100, so may find this difficult. They would not be expected to do this calculation in maths without a calculator.

When students can be expected to use a multiplier.

Finding original amount calculations.

Percentage change

Foundation level students learn this in unit 14 of Pearson GCSE, i.e. Autumn term of year 11.

Approach

Convert a percentage to a fraction

Worked example

Write 20% as a fraction.

$$20\% = \frac{20}{100} \xrightarrow{\div 10} \frac{2}{10} \xrightarrow{\div 2} \frac{1}{5}$$

$$20\% = \frac{20}{100} = \frac{2}{10} = \frac{1}{5}$$

First write 20% as a fraction of 100.

Then simplify the fraction by dividing the numerator and denominator by the same number. Keep doing this until the fraction is in its simplest form.

Convert a percentage to a decimal

Worked example

Write 35% as a decimal.

$$35\% = \frac{35}{100} = 0.35$$

Write 35% as a fraction out of 100. Then divide 35 by 100 to write it as a decimal.

Convert a fraction to a percentage

Convert the fraction to a decimal, then convert the decimal to a percentage.

For example: $\frac{34}{80} = 0.425 = 42.5\%$

Students can input $\frac{34}{80}$ as a fraction into a scientific calculator and press = (or the S-D button on some calculators) to get the equivalent decimal.

Write one number as a percentage of another

Write as a fraction, then convert to a percentage.

For example, in a class of 28 students, 13 are boys. What percentage are boys?

$\frac{13}{28} = 0.4642\ldots = 46.4$ (1 dp)

Without a calculator:

$$\text{Percentage of magnesium} = \frac{\text{mass of magnesium}}{\text{total mass}} \times 100$$

$$= \frac{\overset{3}{\cancel{24}}}{\underset{\underset{1}{\cancel{8}}}{\cancel{40}}} \times \overset{20}{\cancel{100}}$$

$$= 60\%$$

Calculating a percentage of an amount

50% is the same as $\frac{1}{2}$, so to find 50% divide by 2.

10% is the same as $\frac{1}{10}$, so to find 10% divide by 10.

To calculate e.g. 30% mentally, you can find 10% and multiply by 3.

To calculate 5% mentally, find 10% and halve.

Calculating percentages using a calculator

Input the percentage as a fraction

For example, to calculate 30% of 20 m, input $\frac{30}{100} \times 20$ and press = to get 6 m.

Input the percentage using a decimal multiplier

65% = 0.65

So to calculate 65% of 80 kg, input 0.65 × 80 and press = to get 52 kg.

Percentage increase/decrease

Work out the increase and add it on/subtract it

Examples

To increase 45 g by 20%

20% of 45 g = 9 g

45 + 9 = 54 g

To decrease 220 ml by 5%

5% of 220 ml = 11 ml

220 – 11 = 209 ml

Using a multiplier

Examples

To increase 45 g by 20%

After the increase you will have 100% + 20% = 120% = 1.2

1.2 × 45 = 54 g

To decrease 220 ml by 5%

After the decrease you will have 100% – 5% = 95%

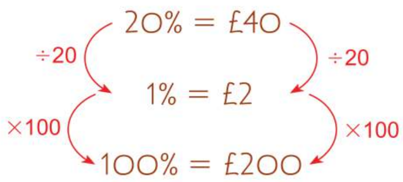
0.95 × 220 = 209 ml

Finding original amount

Using arrow diagrams

Worked example

20% of an amount is £40.
Work out the original amount.



Using function machines

Example

The mass of a substance reduces by 15%.

The final mass is 320 g.

Calculate the original mass.

To calculate a mass after 15% decrease, you would multiply by 0.85:

original $\rightarrow$ $\boxed{\times\ 0.85}$ $\rightarrow$ 320 g

376.5 g $\leftarrow$ $\boxed{\div\ 0.85}$ $\leftarrow$ 320 g

Percentage change

percentage change $= \frac{\text{actual change}}{\text{original amount}} \times 100$

Example

In 2010 a box of tissues cost 80p.

In 2014 a similar box cost £1.20.

The actual increase in price is 120 – 80 = 40p.

The fractional increase is $\frac{\text{actual increase}}{\text{original price}} = \frac{40}{80}$

$\frac{40}{80}$ as decimal is 0.5

Percentage increase is 0.5 × 100 = 50%

Terminology

- Percent means 'out of 100'. A percentage is a fraction with denominator 100.
- You can calculate percentages of amounts, e.g. 20% of 500 g.
- You can write one number as a percentage of another, e.g. write $\frac{7}{50}$ as a percentage.

3.3 Converting between fractions, decimals and percentages

Demand

All students will learn how to convert between fractions, decimals and percentages in KS3.

Approach

Percentage to fraction to decimal

$40\% = \frac{40}{100} = 0.4$

Decimal to percentage

Multiply by 100:

$0.3 = 30\%$

$0.02 = 2\%$

Percentage to decimal

Divide by 100:

$62\% = 0.62$

$7.5\% = 0.075$

Simple fractions to percentages

Multiply or divide both numbers to get a fraction with denominator 100:

$$\frac{3}{20} \underset{\times 5}{\overset{\times 5}{=}} \frac{15}{100} = 15\%$$

$$\frac{62}{200} \underset{\div 2}{\overset{\div 2}{=}} \frac{31}{100} = 31\%$$

Convert a fraction to a percentage

Convert the fraction to a decimal, then convert the decimal to a percentage.

For example:

$\frac{34}{80} = 0.425 = 42.5\%$

Students can input $\frac{34}{80}$ as a fraction into a scientific calculator and press = (or the S-D button on some calculators) to get the equivalent decimal.

Terminology

- When converting decimals to percentages or vice versa, do not say 'move the decimal point two places'. Instead, say 'multiply by 100' or 'divide by 100' as appropriate.

  $0.52 = 52\%$ $\qquad$ $3\% = 0.03$

3.4 Ratios

Demand

Students learn to simplify ratios, and write them in the form 1 : *n* or *n* : 1 in KS3.

Students learn to relate ratios to fractions in KS3, but many continue to make errors with this type of calculation.

Approach

Simplifying ratios

A ratio in its simplest form only contains whole number values.

Divide all the numbers in the ratio by the highest common factor:

$$\begin{array}{ccc} & 2 : 6 & \\ \div 2 & & \div 2 \\ & 1 : 3 & \end{array}$$

$$\begin{array}{ccc} & 6 : 15 & \\ \div 3 & & \div 3 \\ & 2 : 5 & \end{array}$$

This ratio is not in its simplest form, because the two numbers both still have a common factor, 2:

$$\begin{array}{ccc} & 8 : 20 & \\ \div 2 & & \div 2 \\ & 4 : 10 & \end{array}$$

Writing in the form 1 : n (sometimes called a unit ratio)

Divide both numbers by the first number in the ratio:

$$\begin{array}{ccc} & 5 : 7 & \\ \div 5 & & \div 5 \\ & 1 : 1.4 & \end{array}$$

Writing in the form n : 1

Divide both numbers by the second number in the ratio:

$$\begin{array}{ccc} & 20 : 12 & \\ \div 12 & & \div 12 \\ & 1.67 : 1 & \end{array}$$

Comparing ratios

Write both ratios in the form 1 : *n* or *n* : 1.

Example

Ben makes a drink with 20 ml squash to 120 ml water.

Fiona makes a drink with 15 ml squash to 85 ml water.

Whose squash is stronger?

Ben

$$20 : 120 \xrightarrow{\div 20} 1 : 6$$

Fiona

$$15 : 85 \xrightarrow{\div 15} 1 : 5.7$$

Fiona's drink has less water for 1 ml squash, and so is stronger.

Ratio and proportion

A mixture is made from two liquids A and B in the ratio 2 : 3.

What fraction of the mixture is

a) liquid A?

b) liquid B?

Draw a bar model to illustrate the mixture:

A	**A**	**B**	**B**	**B**

$\frac{2}{5}$ is A and $\frac{3}{5}$ is B

Terminology

Write the ratio of *A* to *B* means write *A* : *B*. If you want students to write the ratio as $\frac{A}{B}$ you need to say 'write the ratio as $\frac{A}{B}$'.

To simplify a ratio, divide all the numbers in the ratio by their highest common factor.

To compare ratios, write them in the form 1 : *n*, or n : 1. This is sometimes called a unit ratio.

A **ratio** compares two quantities, and translates into a statement such as 'for every 3 black there are 2 red'.

A **proportion** compares a part with a whole. A proportion can be given as a fraction or a percentage.

Common error

Students look at 2 : 3 and think the fraction is $\frac{2}{3}$.

4. Probability

Demand

All students should learn how to calculate probabilities for single events in KS3.

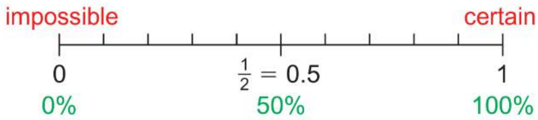
Calculating probabilities of combined events in covered in GCSE Maths, by drawing sample space diagrams or probability trees.

The Appendix listing mathematical skills required for GCSE Science specifies Simple probability, so it is unlikely that probability of combined events will be required.

Approach

Terminology

- The probability of an impossible event is zero. The probability of a certain event is 1.
- You can show probabilities on a probability scale:



- Probabilities can be expressed in fractions, decimals and percentages. All probabilities have a value between 0 and 1.
- Probability of an event happening = $\frac{\text{number of succesful outcomes}}{\text{number of possible outcomes}}$.
  In maths we expect students to give probabilities in the most convenient/accurate way for the data, rather than asking for all versions. e.g. for a dice we expect students to write the probability of rolling a 6 as $\frac{1}{6}$, not as decimal which would be 0.166666… and require rounding, or as a percentage 16.666…. which would require rounding.
- For probability, fractions do not have to be written in their simplest form. Therefore the probability of rolling an even number on a dice can be given as $\frac{3}{6}$, and does not have to be simplified to $\frac{1}{2}$ (though it may be).
- To find the probability of an event not happening, subtract the probability of it happening from 1.
- P(event does not happen) = 1 – P(event happens).
- For percentage probabilities: P(event does not happen) = 100% – P(event happens).
- **Theoretical probability** is calculated based on equally likely outcomes. e.g. for a fair dice, there are 6 equally likely outcomes, so the probability of each outcome is $\frac{1}{6}$. For a fair coin, there are 2 equally likely outcomes, so probability of head = $\frac{1}{2}$.
- **Experimental probability** is an estimate calculated from experimental data. For example in an experiment, buttered toast is dropped 100 times and lands butter side down 61 times. The experimental probability of toast dropping butter side down is $\frac{61}{100}$.
- For experimental probability, the larger the number of trials the more reliable the estimate of probability. e.g. if you drop the toast 1000 times, you will get a more accurate estimate of the probability of it dropping butter side down.
- Be careful with question wording. If you ask: 'Calculate the probability of someone in your class having brown hair' the answer is probably 1 because there is probably someone in the class who has brown hair. More accurately, say: 'A person from your class is picked at random. What is the probability that she or he has brown hair?'

- Similarly, this type of question is inaccurate: 'How does the probability of a shopper choosing a blue T shirt compare with the probability of choosing a red T-shirt?' Each shopper has his/her own probability. Instead ask: Is a shopper picked at random more likely to choose a blue or a red T-shirt?
- For 'event not happening' type of questions, e.g. in the context of picking a diamond from a pack of cards, asking 'what is the probability of not picking a diamond' is not strictly correct, as it includes the possibility/probability of not picking a card at all. The correct wording is 'what is the probability of picking a card that is not a diamond?'

Common error – an example

Students collected numbers of people with different coloured hair/eyes and calculated the experimental probability of each, for their class.

They were then asked to calculate the experimental probability of a student picked at random having red hair and blue eyes.

As hair and eye colour are not independent of each other, the only way to do this would be to collect the data in hair/eye pairs and calculate the experimental probability of each combination. You cannot do it by multiplying the two probabilities together – this only works when events are independent.

5. Algebraic symbols

Demand

Only top sets maths students will have used the 'proportional to' symbol $\propto$ at KS3.

All students will learn to use it in GCSE Maths.

Terminology

- $<$ means less than and $>$ means greater than. $\leq$ means less than or equal to and $\geq$ means greater than or equal to. Students use this notation particularly in grouped frequency tables.
- Students should all be familiar with the equals sign, =
- **Inconsistency:** In maths we use $\approx$ to mean 'is approximately equal to'.
  The symbol ~ is used in probability distributions in AS level maths.
- The symbol $\propto$ means 'is proportional to'.

6. Equations and Formulae

Demand

All students should be able to substitute into a simple word formula or formulae using letters at the end of KS3.

Students may need help with the units at first, as there is not much emphasis on this in GCSE maths. Just putting in numbers without considering units is not good practice.

At KS3, top and middle sets students learn to change the subject of simple formulae such as $F = ma$, $D = \frac{m}{v}$,. $F = T + R$, $F = T - mg$. Lower ability students only rearrange formulae such as $F = ma$ and $D = \frac{m}{v}$.

Students learn to change the subject of formulae such as $\frac{L}{E} = \frac{d_E}{d_L}$, or formulae involving powers and roots in GCSE Maths.

Approach

Using function machines to solve equations

Function machines are a pictorial way of representing 'what happens' in an equation. Working backwards through the function machine solves the equation.

Students learn this approach first, before moving on to the 'balancing' method shown below.

Example 2

Solve the equation $3a + 7 = 13$

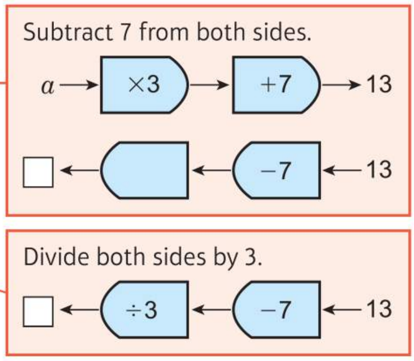
$3a + 7 - 7 = 13 - 7$

$3a = 6$

$\frac{3a}{3} = \frac{6}{3}$

$a = 2$

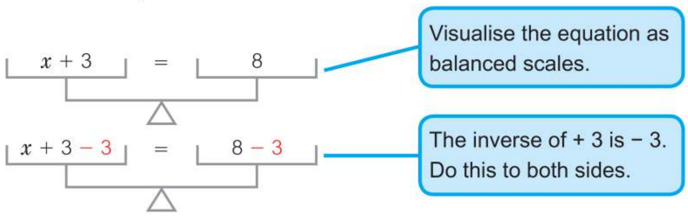
Check: $3a + 7 = 3 \times 2 + 7 = 13$ ✓



Using inverse operations to solve equations – balancing

Students learn this method once they are used to the idea of inverse operations, from working backwards through function machines. After seeing one or two examples like this, they will solve equations without drawing in the 'scales'.

Solve the equation $x + 3 = 8$.



$x = 8 - 3$

$x = 5$ Simplify both sides to find x.

Check: $x + 3 = 5 + 3 = 8$ ✓

The emphasis is on 'doing the same' to both sides, to keep the scales balanced.

Example

Solve $2x - 15 = -3$

Add 15 to both sides: $2x = 12$

Divide both sides by 2: $x = 6$

Changing the subject of a formula using function machines

Using the same approach as for simple equations, above.

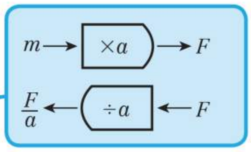
Worked example

Make m the subject of the formula $F = ma$.

$F = m \times a$

$\frac{F}{a} = m$

or $m = \frac{F}{a}$

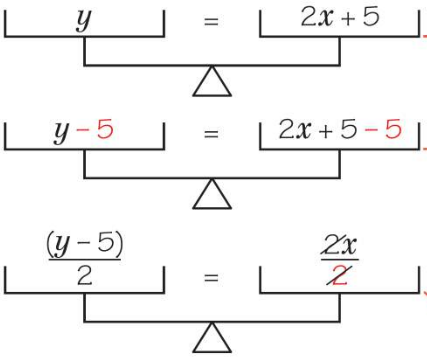


Changing the subject of a formula using inverse operations – balancing

Using the same approach as in learning how to solve equations, as above.

Example 6

Rearrange $y = 2x + 5$ to make x the **subject** of the formula.



Use the balancing method.

The inverse of +5 is −5. Do this to both sides.

Communication hint
The subject of a formula is the letter on its own, on one side of the equals sign.

The inverse of ×2 is ÷2. Do this to both sides.

$x = \frac{y-5}{2}$ ✓

Students then move on to rearranging using the balancing method, without drawing in the scales.

Example 2

a Make a the subject of the formula $v^2 = u^2 + 2as$

b Make x the subject of the formula $y = \frac{ax+b}{c}$

a $v^2 = u^2 + 2as$

$v^2 - u^2 = 2as$ — Subtract u^2 from both sides.

$\frac{v^2 - u^2}{2s} = a$ — Divide both sides by $2s$.

$a = \frac{v^2 - u^2}{2s}$ — Re-write in the form $a = \ldots$

b $y = \frac{ax+b}{c}$

$cy = ax + b$ — Multiply both sides by c.

$cy - b = ax$ — Subtract b from both sides.

$\frac{cy-b}{a} = x$ — Divide both sides by a.

$x = \frac{cy-b}{a}$ — Re-write in the form $x = \ldots$

Terminology

- **<u>Inconsistency</u>:** The list of mathematical skills required for science includes: change the subject of an equation. In maths we distinguish between equations and formulae, and only formulae have subjects.
- The subject of a formula is the letter on its own, on one side of the equals sign.
- A formula is a rule that shows the relationship between two or more variables (letters). You can use substitution to find an unknown value. So $S = \frac{d}{t}$ or $F = ma$ or $v = u + at$ are formulae.
- An equation can be solved to find the value of the letter. Usually this means there is only one letter, as in $2x + 5 = 14$ or $3x = 12$. (In simultaneous equations you can solve them both together to find the values of two variables, e.g. $2x + y = 5$ and $x - 3y = 10$)
- **<u>Inconsistency</u>:** In maths, students are taught to give any non-integer solutions to equations as fractions, not decimals, as fractions are more accurate. For example, the solution to 3x = 5 is x = $\frac{5}{3}$.
- Substituting into a formula can give you an equation to solve. E.g. $S = \frac{d}{t}$, when S = 6 and t = 3, $6 = \frac{d}{3}$, which you can solve to get $d = 18$.
- Letters representing variables should be in italics.
- In word formulae and in formulae using letters, show division as a fraction, not using ÷ e.g. speed = $\frac{\text{distance}}{\text{time}}$ and $S = \frac{d}{t}$, not $S = d \div t$
- $F = ma$ means $F = m \times a$, but it is not necessary to write the multiplication sign.
- In maths we show them formulae triangles for speed and density formulae. But we use a more general approach to solving equations and rearranging formulae, using inverse operations.
- Inconsistency: In maths, to solve equations and change the subject of a formula we use inverse operations and the idea of balancing equations by doing the same to both sides (see examples below). We do not say 'cross multiply' or 'move through the equals and change the sign', or similar explanations.
- **<u>Inconsistency</u>:** In maths, we use 'average speed' = $\frac{\text{total distance}}{\text{total time}}$ we don't call it 'mean speed'.
- **<u>Inconsistency</u>:** In maths we use letters to represent variables and sometimes constants. These may be Greek letters. We do not call any of them symbols.

Substituting into formulae using the correct units for physical quantities

- In maths students often substitute into expressions or formulae without thinking about units, e.g. to find the points on the line $y = 3x + 2$. In science they may not consider units unless this is pointed out to them.
- When a formula is given, the units of each quantity should be given with it. If students are expected to use the formula to calculate e.g. speed in m/s when they are given the distance in cm, they need to be shown (at first) that they need to convert the distance into metres first.
- It would be helpful to show several different possibilities for quantities that can be measured in different units, e.g. for speed: distance in m, time in s gives m/s, distance in km, speed in h gives km/h, etc.
- In general, converting measures into different units before substituting them into the formula is easier than converting the units of the final answer. (See also Units section of this document.)

7. Calculations (including estimation and multiplicative relationships)

7.1 Multiplicative relationships

Demand

All students learn how to add, subtract, multiply and divide positive and negative integers, decimals and fractions at KS3.

Many students find it difficult to know whether to multiply or divide in calculations, and a more pictorial approach, e.g. function machines and arrow diagrams, can help to make that clear.

Approach

Using function machines and arrow diagrams

These are two ways of representing a calculation more visually, to aid understanding and help identify whether to multiply or divide.

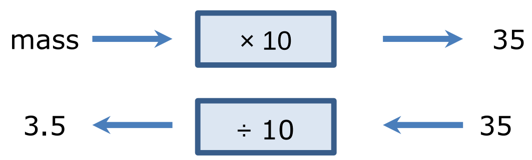
Example

To find the weight of a particular mass, multiply the mass by 10.

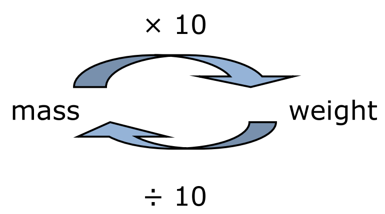
An object weighs 35 N. What is its mass?

Draw the function machine to represent what has happened to the mass.

Work backwards through it to find the original mass.



or you could use an arrow diagram like this:



Multiply by decimals

Estimate the answer, by rounding all values to 1 sf.

Remove all the decimal points and work out the multiplication.

Use your estimation to decide where to put the decimal point in the answer.

Worked example

Work out 5.6 × 4

Use an estimate to check your answer.

$$\begin{array}{r} 5\ 6 \\ \times \quad 4 \\ \hline 2\ 2\ 4 \\ \hline {\scriptstyle 2} \end{array}$$

56 × 4 = 224

5.6 × 4 = 22.4

Check: 5.6 ≈ 6, 6 × 4 = 24

22.4 is close to 24

Ignore the decimal point and work out 56 × 4. 56 ÷ 10 = 5.6, so work out 224 ÷ 10 to get the final answer.

Divide decimals by whole numbers

Worked example

Work out 73.5 ÷ 3

$$\begin{array}{r} 2\ 4\ .\ 5 \\ 3\overline{)7\ {}^{1}3\ .\ {}^{1}5} \end{array}$$

73.5 ÷ 3 = 24.5

First write the decimal point for the answer above the decimal point in the question. Then divide as normal, starting from the left.

Divide by decimals

To divide by a decimal, multiply both numbers by a power of 10 (10, 100, …) until you have a whole number to divide by.

Then work out the division.

Worked example

Work out 67.8 ÷ 1.2

$$1.2\overline{)67.8} \xrightarrow{\times 10} 12\overline{)678}$$

1.2 has one decimal place, so multiply both numbers by 10.

$$\begin{array}{r} 5\,6.\,5 \\ 12\overline{)6\,7\,{}^{7}8.{}^{6}0} \end{array}$$

Work out the division.

Check: 12 × 56.5 ≈ 10 × 60 = 600

Terminology

- ‘Sum’ means 'add' – all others are 'calculations'. Please use correct terminology to avoid confusion in maths.
- Using a calculator – as all calculators vary, in maths books we do not tell them which keys to press. Instead we say ‘make sure you know how to input fractions calculations on your calculator’.
- Students should use a scientific calculator and so be able to input fractions and calculations exactly as they are written on the page.
- Students should be able to calculate powers and roots using a calculator. They should know the squares of integers up to 12 and their related square roots.
- Multiplying and dividing by 10, 100, 1000: Avoid telling them to ‘move the decimal point’. The position of the decimal point is fixed.
- Also avoid telling them to ‘add a zero’ when multiplying by 10. This does not work for e.g. 3.4 × 10 ≠ 3.40
  To multiply by 10, move the digits one place to the left.
  42 × 10 = 420
  To multiply by 100, move the digits two places to the left, and so on.
  3.6 × 100 = 360

- To divide by 10, move the digits one place to the right.
  $42 \div 10 = 4.2$
  To divide by 100, move the digits two places to the right, and so on.
  $5700 \div 100 = 57$
- Avoid giving students a rule for a calculation, e.g. 'multiply by 10' without explaining the working out.

7.2 Estimation

Demand

All students learn to estimate answers to calculations in GCSE Maths, but lower ability students will not be able to estimate answers to calculations involving powers and roots.

Approach

Estimate the answer to $591 \times \frac{97}{289}$

Rounding each of the numbers to 1 significant figure gives $600 \times \frac{100}{300}$

So a good estimate would be 200.

Terminology

- Avoid any suggestion that an estimate involves guessing. The calculation is not 'rough' – it is accurate, but the numbers you use or the assumptions you make are estimates or approximations.
- To estimate the result of a calculation in maths, we round all values to 1 significant figure.
- In calculations involving division or square roots, you can round one or more values to give a 'nice' division or root.
  e.g. to estimate $\frac{3.7 \times 7.5}{4.8}$, rounding the values on the top to 1 sf gives $4 \times 8 = 32$.
- So approximating 4.8 to 4 gives the calculation $32 \div 4 = 8$.
- Use the symbol $\approx$ to show the estimated answer to a calculation.

7.3 Order of magnitude calculations

Approach

- Making order of magnitude calculations is not taught in GCSE Maths. Please see notes on dividing with decimals above.
- In maths students use and manipulate formulae, but do not meet the magnification formula:
  $\text{magnification} = \frac{\text{size of image}}{\text{size of real object}}$

For notes on substituting into formulae and changing the subject, see Section 6 Equations and Formulae.

8. Use angular measures in degrees

Demand

All students learn to measure angles in degrees in KS3, and to calculate missing angles in simple diagrams.

Terminology

- Angles around a point add to 360°, angles on a straight line add to 180°.
- A right angle is 90°.
- Perpendicular lines meet at 90°.
- Students can use a circular protractor to measure angles greater than 180°.
- Angles less than 90° are acute, angles between 90° and 180° are obtuse, angles greater than 180° are reflex angles.

9. Visualise and represent 2-D and 3-D forms, including two-dimensional representations of 3-D objects

Demand

All students learn about simple nets in KS3.

Terminology

- 2-D representations of 3-D shapes include 3-D sketches, accurate 3-D drawings on isometric paper, nets and plans and elevations.
- A net is the 2-D shape that can be folded up to make a 3-D shape.
- The plan view is the view from above an object. The side elevation is the view from one side and the front elevation is the view from the front.

10. Calculate areas of triangles and rectangles, surface areas and volumes of cubes

Demand

All students learn to calculate the area of a rectangle and triangle in KS3.

Top and middle sets maths students use hectares in KS3.

All students calculate surface area and volume of cubes and cuboids in KS3.

Approach

Estimating area of irregular shape – counting squares

To calculate the area of an irregular shape, such as a leaf, draw round the shape on graph paper that has small squares. Then count the squares inside the area. For squares that cross the perimeter, count those that are more than half-in as whole ones, and don't count those that are more than half-out. This will give you an **estimate** of the area – the smaller the graph squares, the more accurate the estimate.

Remember that area is measured in square units, such as m^2 or cm^2.

Calculate area of rectangle

Area of a rectangle = length × width

$A = l \times w$ or $A = lw$

Calculate area of triangle

This formula works for all triangles, not just right-angled ones.

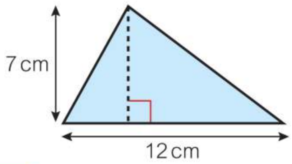
h is the perpendicular height.

Area of triangle = $\frac{1}{2}$ × base × height

$A = \frac{1}{2}bh$

Worked example

Work out the area of this triangle.



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 12 \times 7$

$= 42\ \text{cm}^2$

Write the formula, then substitute the numbers into the formula.

Key point

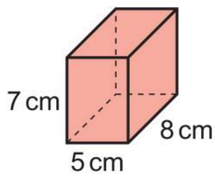
Area of a triangle = $\frac{1}{2}$ × **base** length × **perpendicular height** which can be written as $A = \frac{1}{2}bh$. The height measurement must be perpendicular (at 90°) to the base.

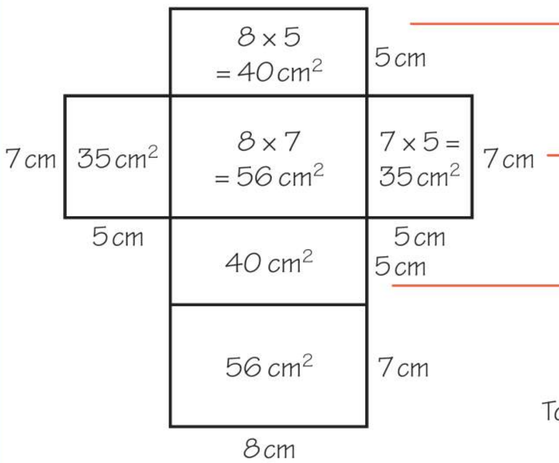
In a right-angled triangle, the two sides that meet at the right angle are the base and the height.

Calculate surface area by drawing the net

Example 4

Work out the surface area of this cuboid.





Sketch the net.

Label the lengths.

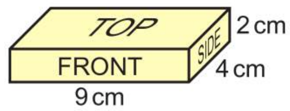
Work out the area of each face.

Total surface area = 40 + 56 + 40 + 56 + 35 + 35
= 262 cm²

Calculate surface area without drawing the net

Use the fact that opposite faces are equal – this question illustrates the steps.

Reasoning



a i What is the area of the top of this cuboid?
ii Which other face of the cuboid is identical to this one?

b i What shape is the front of this cuboid?
ii Which other face of the cuboid is identical to this one?

c i What shape is the side of this cuboid?
ii Which other face of the cuboid is identical to this one?

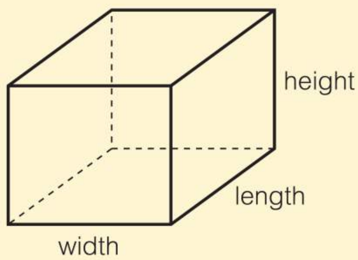
d Copy and complete this calculation to work out the total surface area of the cuboid.

$2 \times \square + 2 \times \square + 2 \times \square = \square \text{ cm}^2$

Volume of cuboid

Key point

Volume of a cuboid
= length × width × height
= $l \times w \times h = lwh$



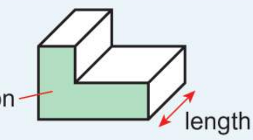
Common error

Students use length × width × height to calculate the volume of **any** solid. Please state clearly in questions that the solid concerned is a cuboid (or for example, the sample can be modelled on a cuboid) to avoid reinforcing this misconception.

Volume of a prism

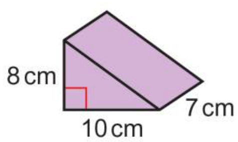
Key point 12

Volume of a prism = area of cross-section × length



Example 5

Work out the volume of this prism.



Volume = area of cross-section × length — Write down the formula.

Area of triangle $= \frac{1}{2} \times 10 \times 8$ — Work out the area of the cross-section.

$= 5 \times 8$

$= 40$

Volume $= 40 \times 7$ — Substitute the area of the cross-section and the length into the formula.

$= 280 \text{ cm}^3$ — Write the units.

Terminology

- Use the mathematical terms rectangle and rectangular (instead of oblong) and rhombus (instead of diamond).
- Use the mathematical term cuboid (instead of box-like).
- Specify the shape of an object when you ask students to calculate the area. If it is a rectangle, say so clearly. Otherwise this reinforces a common misconception that the area of any shape is length × width. For example, 'Estimate the abundance of crabs in an area 60 m long and 10 m wide' is not accurate enough. Tell them it is a rectangular area.
- The perimeter of a 2-D shape is the distance all around the outside.
- The area of a 2-D shape is the amount of space inside the shape. It is measured in squared units mm^2, cm^2, m^2, hectares (1 ha = 10 000 m^2) and km^2.
- You can estimate the area of an irregular shape by drawing around it on cm squared paper and counting the squares.
- If a shape is close to a rectangle, you can estimate the area by approximating it to a rectangle.
- The surface area of a 3-D shape is the total area of all the surfaces added together.
- In maths we use 'area' for 2-D shapes (e.g. tennis courts) and 'surface area' for 3-D shapes, because it is the areas of all the surfaces added together.
- To calculate the surface area of cuboid, find the areas of all the faces and add them together.
- **Inconsistency:** In maths we do not give students a formula for the surface area of a cuboid.
- The volume of a 3-D shape is the amount of space it takes up. It is measured in cubed units, mm^3, cm^3, m^3.
- Capacity is the amount of liquid a 3-D solid can hold. It is measured in ml or litres.

11. Calculate circumference and area of circles, volumes of cylinders and spheres, surface areas of cylinders and spheres

Demand

Students calculate circumference and area of circles at KS3.

Students do not calculate volume and surface area of spheres and cylinders until GCSE maths.

Many students find calculating the surface area of a cylinder extremely difficult, even at GCSE. I would expect to spend a whole maths lesson on it.

Approach

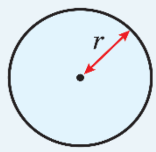
Circumference and radius of circle

Key point 2

The Greek letter π (pronounced pi) is the ratio of the circumference of a circle to the diameter. Its decimal value never ends, but starts as 3.1415926535897…

The formula for the circumference of a circle is $C = \pi d$.

If you know the radius you can use $C = 2\pi r$.



Example 1

The circumference of a circle is 60.8 cm. Work out the radius of the circle.

$C = 2\pi r$

$60.8 = 2\pi r$ — Substitute the values that you know.

$\frac{60.8}{2\pi} = r$ — Rearrange to make r the subject.

$r = 9.676\,620\,538$ — Enter $\frac{60.8}{\pi}$ as a fraction on your calculator.

radius = 9.7 cm (to 1 d.p.) — Write the answer to the same degree of accuracy as the measurement given. Remember to include the units.

Area of a circle

Key point 4

The formula for the area A of a circle with radius r is $A = \pi r^2$.

Example 2

A circle has a radius of 6.4 cm.

Work out the area of the circle. Give your answer correct to 3 significant figures (3 s.f.).

$A = \pi r^2$

$= \pi \times 6.4^2$ — Write the substitution. Input it into your calculator.

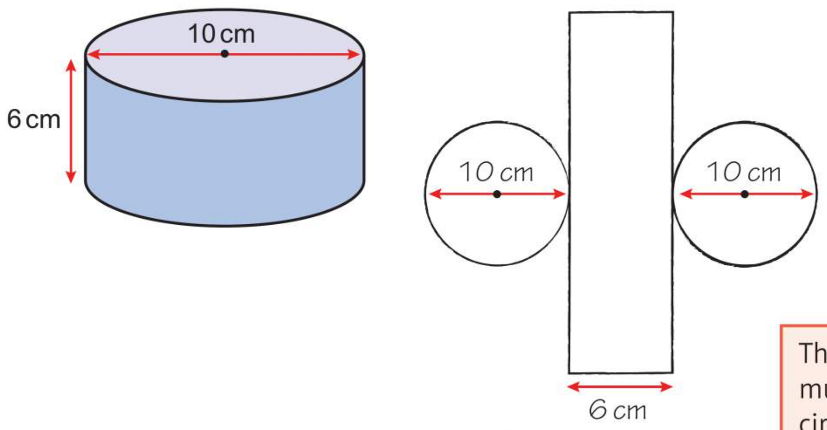
$= 128.6796351$ — Write down all the figures on the calculator display.

$= 129\text{ cm}^2$ (to 3 s.f.) — Round the answer to the required accuracy. Remember the units.

Surface area of cylinder using a net

Example 6

Work out the surface area of this cylinder.



Sketch the net.

The length of the rectangle must be the same as the circumference of the circle.

Circle area $= \pi \times 5^2 = 78.539\,816\,34\text{ cm}^2$

Circumference of circle $= \pi \times 10 = 31.415\,926\,54\text{ cm}^2$

Rectangle area $= 6 \times 31.41592654 = 188.495\,559\,2\text{ cm}$ — 6 × circumference of circle

Total surface area of cylinder = 2 × circle area + rectangle area

$= 2 \times 78.539\,816\,34 + 188.495\,559\,2$

$= 345.575\,191\,9$

$= 345.6\text{ cm}^2$ (to 1 d.p.)

Use all the digits in the calculation.

Round the final answer to a suitable level of accuracy.

Volume of a cylinder

Volume of prism = area of end face × height

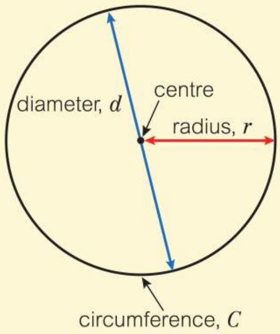
Volume of a cylinder = area of circular end face × height ($V = \pi r^2 h$)

Terminology

- The radius of a circle is the distance from the centre to the outside edge. The diameter is the distance from one side of the circle to another, through the centre.
- The circumference of a circle is the distance all around the outside.
- There are two formulae for circumference of a circle: πd and $2\pi r$
- The formula for area of a circle is πr^2.
- Students use the π key on their calculators, instead of inputting the rounded value 3.14.
- If a shape is close to a circle, you can estimate its area by approximating it to a circle.
- Formulae: surface area of sphere $4\pi r^2$, volume of sphere $\frac{4}{3}\pi r^3$; volume of cylinder $\pi r^2 h$; surface area of cylinder $2\pi r^2 + 2\pi rh$.
- Surface area to volume ratio $\frac{SA}{V}$. Students do not learn this in mathematics.

Key point

The **circumference** (C) is the perimeter of a circle.
The **centre** of a circle is marked using a dot.
The **radius** (r) is the distance from the centre to the circumference.
The plural of radius is **radii**.
The **diameter** (d) is a line from one edge to another through the centre.



12. Units and compound units, including conversion between units

Demand

All students meet the prefixes for metric units in GCSE Maths. The only ones they are likely to use frequency in maths are kg, km, cm, ml, mm.

All students learn to convert between metric units of area and volume in GCSE Maths.

All students learn to convert speeds in m/s to km/h and vice versa in GCSE Maths.

Students are not expected to know metric/imperial conversions, or imperial to imperial conversions such as lbs to stones, or feet to yards.

Approach

Use arrow diagrams and function machines for simple conversions

‘By counting the number in 15 seconds and multiplying by 4'

Rather than present this as a 'magic' formula, it would be good to get students to work out how many 15 seconds there are in a minute, and so what to multiply by.

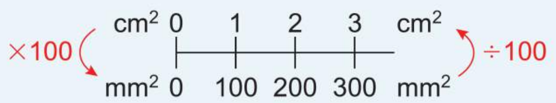
Area conversions

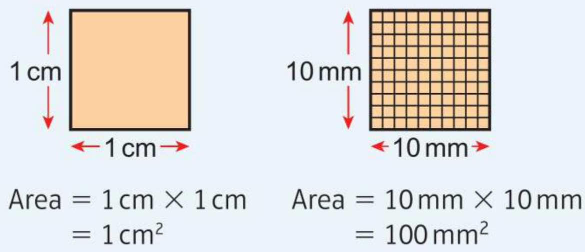
Students may not remember area conversion factors, but will learn in GCSE Maths how to work them out, as follows.

You can use a double number line to convert between area measures.

Key point 4

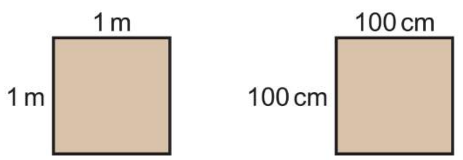
These two squares have the same area.
To convert from cm^2 to mm^2, multiply by 100.
To convert from mm^2 to cm^2, divide by 100.



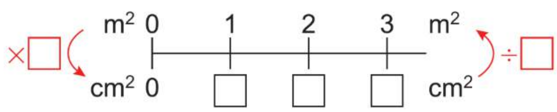


This question shows students how to convert km^2 to m^2 and vice versa.

Use these diagrams to help you work out the number of cm^2 in 1 m^2.



Copy and complete the double number line.



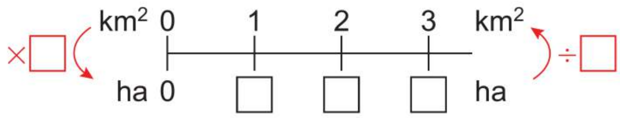
This question shows hectares to km^2 and vice versa.

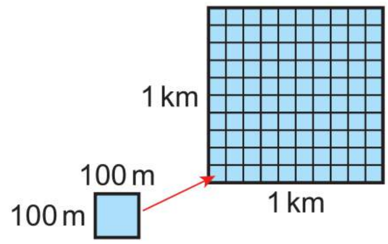
The diagram shows 1 km^2 divided into 100 m squares.

a What is the area of each 100 m square?

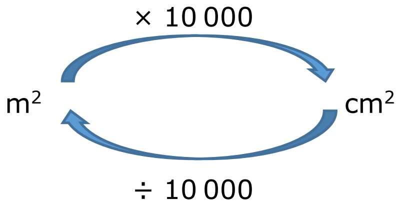
b How many hectares are there in 1 km^2?

c Copy and complete the double number line.





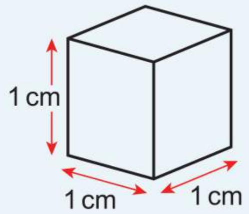
You can use arrow diagrams to help convert between area measures.

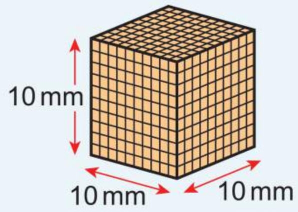


Volume conversions

Key point 13

Volume is measured in mm^3, cm^3 or m^3. These two cubes have the same volume.



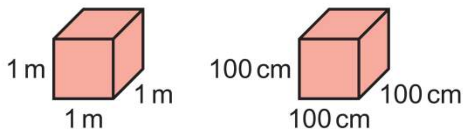


Volume = 1 cm × 1 cm × 1 cm
= 1 cm^3

Volume = 10 mm × 10 mm × 10 mm
= 1000 mm^3

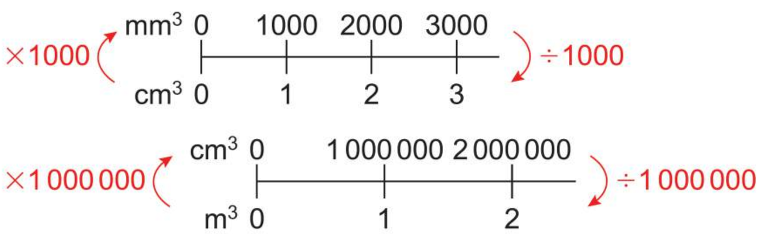
1 cm^3 = 1000 mm^3

6 **a** Use these diagrams to help you work out the number of cm^3 in 1 m^3.

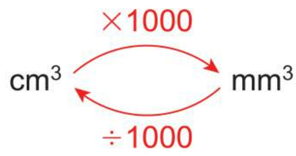


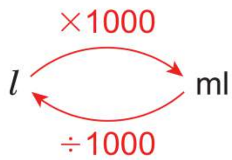
Q6a hint Work out the volume of each cube.

You can use double number lines to help convert between volume measures.



You can use arrow diagrams to help convert between volume measures.





Terminology

- **<u>Inconsistency</u>:** in maths and science we use cm, mm, kg, etc. These are abbreviations, not symbols.
- We use the abbreviations min (not m) for minutes.
- Students do not meet mg and dm^3 in KS3 maths.
- For compound units we use m/s, km/h, g/cm^3, rather than ms^{-1} etc.
- For acceleration we use m/s^2, not m/s/s.
- In our maths books, where we use 'per' we give a literacy hint, e.g. 8 g/cm^2 means 8 grams in every cm^2.
- Area is measured in squared units mm^2, cm^2, m^2, hectares (1 ha = 10 000 m^2) and km^2.
- Volume is measured in cubed units mm^3, cm^3, m^3.
- Capacity is measured in litres and ml. Students do not use dl in maths. Some students may use cl.
- 1 cm^3 = 1 ml
- Some students meet the prefixes for metric units in KS3 maths, e.g. M stands for Mega and means 10^6.
- It is easier to convert measures to the units required before doing a calculation, than to convert the answer into the units required.
- If students are required to convert between metric and imperial units, they should be given the conversion factor. In GCSE Maths they are not expected to know metric/imperial equivalents.
- **<u>Inconsistency</u>:** In maths we call avoirdupois measures imperial measures.

13. Significant figures, decimal places, accuracy

Demand

All students will have learned to round to the nearest whole number and 1, 2 or 3 dp by the end of KS3. They should be able to cope with rounding to more dp as an extension of rounding to 3 dp.

Significant figures

Only Higher tier students learn about upper and lower bounds, and percentage error.

For percentage error they answer questions such as: Given a percentage error of ± 10%, what is the largest/smallest possible value?

Answering questions such as 'What is the percentage error?' for a given value is not covered in the GCSE Maths specification.

Approach

Look at the digit *after the* last one you want to keep. Round up if this digit is 5 or more, round down if it is 4 or less.

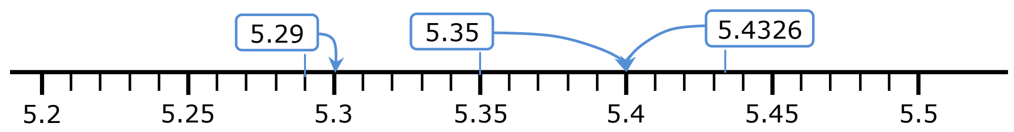
Rounding to 1 dp

5.4|326
less than 5
round down
5.4 (1 d.p.)

5.2|91
5 or more
round up
6.3 (1 d.p.)

5.3|5
5 or more
round up
5.4 (1 d.p.)

On a number line, round to the nearest value with 1 decimal place:



35.42|9
5 or more
round up
35.42 (2 d.p.)

126.37|2
less than 5
round down
126.37 (2 d.p.)

Rounding to 3 dp

0.053|21
less than 5
round down
0.053 (3 d.p.)

11.291|5
5 or more
round up
11.292 (3 d.p.)

Rounding to significant figures

Small numbers

1st significant figure
= 4 ten thousandths

0.000 483

Large numbers

1st significant figure
= 5 billion

518 376 000

Round to 2 significant figures (2 s.f.)

0.00048|3

less than 5
round down

0.00048

51|8 376 000

5 or more
round up

520 000 000

Add zeroes so the 5 is still
in the 'billions' position

Upper and lower bounds calculations

Find the upper and lower bounds of the given values, before doing the calculation.

Terminology

- The number of **decimal places** is the number of digits after the decimal point. So, 10.5219 has 4 decimal places, and 10 has no decimal places.
- In any number the first **significant figure** is the one with the highest place value. It is the first non-zero digit counting from the left. **<u>Inconsistency</u>:** Zero is counted as a significant figure if it is between two other non-zero significant figures. Other zeros are place holders – if you took them out the place value of the other digits would change.

place holders

0 . 0 0 5 0 7 3

1st 2nd 3rd 4th

significant figures

place holders

3 9 0 4 1 2 0 0 0

1st 2nd 3rd 4th 5th 6th

significant figures

- To round a number to a given number of significant figures or decimal places, look at the digit after the last one you need. Round up if the digit is 5 or more, and round down if the digit is 4 or less.
- **<u>Inconsistency</u>:** Rounding numbers reduces accuracy. Your results cannot be more accurate than your starting values. In science: in calculations, your answer cannot have more significant figures than the numbers in the calculation. In maths we tell students not to give more decimal places in the answer than in the calculation, and also to consider if their answers are practical – e.g. could you measure 4.321 cm to

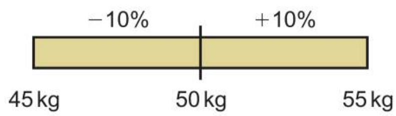
that level of accuracy? In science, where measuring instruments are more accurate, the answer to this may be 'yes'.

- 8.95 rounded to 1 decimal place is 9.0. You must write the '.0' to show the value in the decimal place.
- When a value is rounded, its true value lies within half a unit either side of the rounded value.
  The values of x that round to 5.2 to 1 dp are
  $5.15 \leq x < 5.25$
- Note, this means that the highest value that rounds to 5.2 to 1 dp is 5.2499999….
- The upper bound is half a unit greater than the rounded measurement.
  The lower bound is half a unit less than the rounded measurement.

$$\underbrace{12.5}_{\text{lower bound}} \leq x < \underbrace{13.5}_{\text{upper bound}}$$

NB Only Higher tier maths students learn this.

- To determine an appropriate level of accuracy for an answer to a calculation, you can find the upper and lower bounds of the calculation.
  E.g. if upper bound is 28.42896
  and lower bound is 28.42712
  then 28.42 is a suitable level of accuracy.
- NB Only Higher tier maths students learn this.
- A 10% error interval means that a value could be up to 10% larger or smaller than the value given.



- You can write an error interval as an inequality: $45 \leq m \leq 55$ kg.

14. Standard form

Demand

All students learn to write numbers in index form and use the index laws for multiplication and division in KS3.

All students learn the positive and negative powers of 10 in Unit 1 of GCSE Maths. Foundation students often find the negative and zero powers difficult to understand/remember, as they are the only negative and zero powers they use. Higher students use negative and zero indices with a range of numbers so are likely to have a better understanding.

All students learn to read and write very small and very large numbers in standard form in GCSE.

In Pearson Maths:

- Higher students learn this in the first unit of the GCSE textbook.
- Foundation students learn it in unit 18 of the GCSE textbook, in the spring term of Year 11.

Approach

Calculating powers of 10

Follow a pattern:

$$10^1 = 10$$
$$10^2 = 10 \times 10 = \square$$
$$10^3 = 10 \times 10 \times 10 = \square$$
$$10^4 = 10 \times 10 \times 10 \times 10 = 10\,000$$
$$10^5 = \ldots$$
$$10^6 = \ldots$$

$$10^3 = 1000$$
$$10^2 = 100$$
$$10^1 = 10$$
$$10^{\square} = 1$$
$$10^{-1} = \frac{1}{10} = 0.1$$
$$10^{\square} = \frac{1}{100} = \frac{1}{10^2} = 0.01$$
$$10^{\square} = \square = \square = \square$$

Writing large numbers in standard form

These examples are taken from Pearson GCSE Maths Foundation.

Example 4

Write 4000 in standard form.

$4000 = 4 \times 1000$ — Write the number as a number between 1 and 10 multiplied by a power of 10.

$= 4 \times 10^3$ — Write the power of 10 using indices.

Example 5

Write 45 600 in standard form.

$45\,600 = 4.56 \times 10^4$ — 4.56 lies between 1 and 10. Multiply by the power of 10 needed to give the original number.

4 5 6 0 0

Writing small numbers in standard form

Example 6

Write 0.000 05 in standard form.

$0.000\,05 = 5 \times 0.000\,01$ — Write the number as a number between 1 and 9 multiplied by a power of 10.

$= 5 \times 10^{-5}$

Key point 7

To write a small number in standard form:

- Place the decimal point after the first non-zero digit.
- How many places has this moved the digit? This is the negative power of 10.

Example 7

Write 0.003 52 in standard form.

$0.003\,52 = 3.52 \times 10^{-3}$ — 3.52 lies between 1 and 10. Multiply by the power of 10 needed to give the original number.

0 . 0 0 3 5 2

Calculating with numbers in standard form

Multiplication and division

Example 3

Work out $(5 \times 10^3) \times (7 \times 10^6)$

$5 \times 7 \times 10^3 \times 10^6$ — Rewrite the multiplication grouping the numbers and the powers.

35×10^9 — Simplify using multiplication and the index law $x^m \times x^n = x^{m+n}$. This is not in standard form because 35 is not between 1 and 10.

$35 = 3.5 \times 10^1$ — Write 35 in standard form.

$35 \times 10^9 = 3.5 \times 10^1 \times 10^9 = 3.5 \times 10^{10}$ — Work out the final answer.

Work out $\dfrac{2.4 \times 10^5}{3 \times 10^2}$

Divide 2.4 by 3.
Use the index law $x^m \div x^n = x^{m-n}$ to divide 10^5 by 10^2.

$= 0.8 \times 10^3$

$= 8 \times 10^2$

Addition and subtraction

Write numbers in decimal form before adding and subtracting.

Write the answer in standard form.

Work out $3.6 \times 10^2 + 4.1 \times 10^{-2}$

$= 360 + 0.041$

$= 360.041$

$= 3.60041 \times 10^2$

Work – out $2.5 \times 10^6 - 4 \times 10^4$

$$\begin{array}{r} 2\,500\,000 \\ -\ \underline{\quad 40\,000} \\ 2\,460\,000 \end{array}$$

2.46×10^6

Terminology

- Any number can be raised to a power or index. The power or index tells you how many times the number is multiplied by itself. $3^4 = 3 \times 3 \times 3 \times 3$
- We read 3^4 as '3 to the power 4'.
- Some calculators have a power or index key. In maths we do not tell them which key presses to use, as calculators vary. Instead we would say 'Make sure you know how to input numbers raised to a power on your calculator.'
- Any number raised to the power zero = 1.
- The index laws: To multiply powers of the same number, add the indices
  To divide powers of the same number, subtract the indices.
- Some of the powers of 10:

10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^0	10	10^2	10^3	10^4
0.0001 or $\frac{1}{10\,000}$	0.001 or $\frac{1}{1000}$	0.01 or $\frac{1}{100}$	0.1 or $\frac{1}{10}$	1	10	100	1000	10 000

- Standard form is a way of writing very large or very small numbers as a number between 1 and 10 multiplied by a power of 10.
  $A \times 10^n$ where A is between 1 and 10 and n is the power of 10
- **Inconsistency:** When writing numbers in standard form, do not talk about 'moving the decimal point'. The position of the decimal point remains fixed. Multiplying by a power of 10 moves digits places to the left and dividing by a power of 10 moves digits places to the right.
- On some calculators you can enter numbers in standard form, or answers may be given in standard form. In maths we do not tell them which key presses to use, as calculators vary. Instead we would say 'Make sure you know how to enter and read numbers in standard form on your calculator.'

15. Statistics – average, mean, quartiles/percentiles

Demand

Normal distribution is not covered in Maths GCSE.

Foundation level students do not use quartiles and interquartile range.

Higher level students learn quartiles and interquartile range in Unit 14 of Pearson GCSE maths, i.e. Autumn year 11.

Percentiles are not on the GCSE Maths specification.

Only Higher level students find means and medians from bar charts and histograms in GCSE Maths.

Approach

Calculating the range

From a small data set

Range = largest value minus smallest value

For example, for this data: 2, 2, 5, 7, 2, 4, 6, 9

the range is 9 – 2 = 7

From a frequency table

Range = largest value minus smallest value

Number of eggs	Frequency
1	2
2	15
3	6

Range = 3 – 1 = 2

NB it is the range of the data values, not of the frequencies.

From a grouped frequency table

An estimate of the range is

largest possible value minus smallest possible value

Worked example

In a survey, people were asked their age. The table shows the results.

Age, a (years)	Frequency
$0 \leqslant a < 10$	12
$10 \leqslant a < 20$	15
$20 \leqslant a < 30$	2
$30 \leqslant a < 40$	11

Work out an estimate for the range of ages.

From the frequency table, the smallest possible age is 0 years.

The largest possible age is 40 years.

So an estimate of the range is 40 − 0 = 40 years.

Finding the mode

From a data set

For the data set 2, 2, 5, 7, 2, 4, 6, 9, the mode is 2.

For the data set 1, 1, 3, 4, 2, 5, 3, 3, 2, 2, 1 the modes are 1 and 3.

From a frequency table

For this data, the mode is 2 eggs.

Number of eggs	Frequency
1	2
2	15
3	6

Common error

Students may give '15' as the mode (the highest frequency), rather than 2, which is the number of eggs with the greatest frequency.

From a grouped frequency table

For this data the modal class is $10 \leq a < 20$

Age, a (years)	Frequency
$0 \leqslant a < 10$	12
$10 \leqslant a < 20$	15
$20 \leqslant a < 30$	2
$30 \leqslant a < 40$	11

Finding the median and quartiles

First write the data in order.

0 2 4 5 7 8 10 12 12 12 13 14 14 15 16

The lowest quartile is the 4th value

The median is the 8th value

The upper quartile is the 12th value

Common error

Students may not order the data before finding the median and quartiles.

From a frequency table

Number of eggs	Frequency
1	4
2	8
3	6
4	2
Total	20

There are 20 pieces of data in the table.

The median is the $\frac{20+1}{2}$ = 10.5th data item (i.e. between the 10th and 11th items).

Number of eggs	Frequency
1	4
2	8
3	6
4	2
Total	20

4

4 + 8 = 12

Add up the frequencies to find the 10th and 11th data items.

The 10th and 11th items are both 2 eggs, so the median is 2 eggs.

Calculating the quartiles from a frequency table

The lower quartile is the $\frac{20+1}{4}$ = 5.25th data item (i.e. between the 5th and 6th items).

the upper quartile is the $\frac{3(20+1)}{4}$ = 15.75th data item (i.e. between the 15th and 16th items).

Number of eggs	Frequency
1	4
2	8
3	6
4	2
Total	20

4

4 + 8 = 12

12 + 6 = 18

Add up the frequencies to find the 5th and 6th data items and the 15th and 16th data items.

The 5th and 6th items are both 2 eggs, so the lower quartile is 2 eggs.

The 15th and 16th items are both 3 eggs, so the upper quartile is 3 eggs.

Finding the interval containing the median from a grouped frequency table

Age, a (years)	Frequency
$0 \leqslant a < 10$	12
$10 \leqslant a < 20$	15
$20 \leqslant a < 30$	2
$30 \leqslant a < 40$	11

12

12 + 15 = 27

Add up the frequencies to find the 20th and 21st data items.

Total frequency = 40

Median = $\frac{40+1}{2}$ = 20.5th data item.

The 20th and 21st data items are in the interval $10 \leq a < 20$.

Calculating the mean of a small data set

From Pearson GCSE Maths Foundation.

Example 1

Work out the mean of 3, 6, 7, 7 and 8.

$3 + 6 + 7 + 7 + 8 = 31$

Add the values first to find the total.

$\frac{31}{5} = 6.2$

There are 5 values, so divide the total by 5.

The mean is 6.2

Common error

When using a calculator to calculate a mean, students may add the numbers and not press = before dividing, which will give an incorrect value.

Calculating the mean from a frequency table

From Pearson KS3 Maths.

Worked example

Jack asked students in his class how many pets they had.
Here are his results. Work out the mean.

Number of pets	Frequency	Total number of pets
0	7	$0 \times 7 = 0$
1	8	$1 \times 8 = 8$
2	6	$2 \times 6 = 12$
3	3	$3 \times 3 = 9$
4	1	$4 \times 1 = 4$
Total	25	33

Add a column to the table to work out the total numbers of pets.

Work out the total frequency (number of people in the survey) and the total number of pets.

mean $= 33 \div 25 = 1.32$

mean = total number of pets ÷ number of people

Calculating an estimate of the mean from a grouped frequency table

From Pearson KS3 Maths.

Worked example

In a survey, people were asked their age. The table shows the results.

Age, a (years)	Frequency
$0 \leqslant a < 10$	12
$10 \leqslant a < 20$	15
$20 \leqslant a < 30$	2
$30 \leqslant a < 40$	11

Calculate an estimate for the mean age.

Age, a (years)	Frequency	Midpoint of class	Midpoint × Frequency
$0 \leqslant a < 10$	12	$\frac{0+10}{2} = 5$	$5 \times 12 = 60$
$10 \leqslant a < 20$	15	$\frac{10+20}{2} = 15$	$15 \times 15 = 225$
$20 \leqslant a < 30$	2	25	$25 \times 2 = 50$
$30 \leqslant a < 40$	11	35	$35 \times 11 = 385$
Total	40		720

Add a column to calculate an estimate of the total age for each class.

Calculate the total number of people in the survey and the sum of their ages.

mean = sum of ages ÷ total number of people

$= \frac{720}{40}$

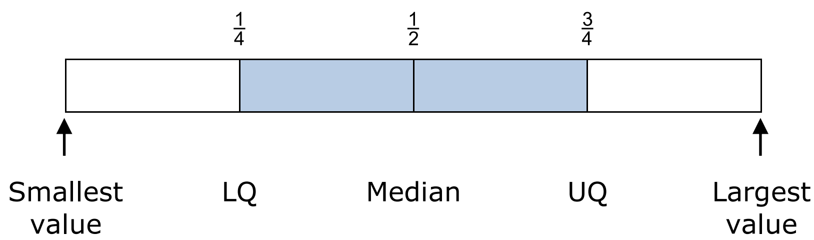
$= 18$

Calculating means and medians from bar charts and histograms

Make a frequency table for the bar chart or histogram, and use the appropriate method shown above.

Terminology

- Mean, median and mode are all averages. In everyday life when someone says 'average' they are usually talking about the mean.
- The range is a measure of spread. It is calculated as largest value − smallest value. Note the range is a single number, not e.g. 3-12, two numbers separated by a hyphen. In a maths question we would say 'Work out' the range – you need to do a calculation to find it.
- A larger range means the data is less consistent. A smaller range means the data is more consistent.
- You can estimate the range from a grouped frequency table, as largest possible value minus smallest possible value.
- Mean = $\frac{\text{total of all the values}}{\text{number of values}}$
- You can calculate the mean from an ungrouped frequency table. For a grouped frequency table, you can calculate an estimate of the mean (because you use the midpoint of each group as an estimate of the data values in that group). Please word such questions as 'Calculate an estimate of the mean'.
- The mode is the most common value. In a frequency table, this is the value with the highest frequency. The mode is one of the data values. A set of data can have more than one mode. For grouped data, the modal class is the class interval with the highest frequency.
- The median is the middle value when the data is written in order. It may not be one of the data values (e.g. it could be halfway between two values).
- For an ordered set of data with an even number of values, the median is the mean of the two middle values (which is the same as the value midway between them).
- For a set of n items of data, the median is the $\frac{n+1}{2}$th data item. When n is very large, you can use the $\frac{n}{2}$th data item.
- If you have an anomalous value (sometimes called an outlier in maths), i.e. one that is likely to have been a recording error, you can ignore this when calculating the mean.
- For a set of ordered data, the median is the value halfway through the data. The lower quartile is the value one quarter of the way into the data set. The upper quartile is the value three quarters of the way into the data set.
- For a set of n items of data, the lower quartile is the $\frac{n+1}{4}$th data item and the upper quartile is the $\frac{3(n+1)}{4}$th data item. When n is very large, you can use the $\frac{3n}{4}$th data item.
- The interquartile range is value calculated by: upper quartile minus lower quartile. It shows how spread out the middle 50% of the data is.



- The 50th percentile is the median. The 10th percentile is the value 10% of the way into the data set, when the data is in order.
- NB Students do not learn about percentiles in GCSE Maths.
- To describe a set of data you should give at least one average and a measure of spread.
- To compare two sets of data you should compare one average and one measure of spread.

16. Sampling and data

Demand

Students learn that interpolation is more accurate than extrapolation, though may not use these terms to describe it.

Students learn to write and criticise questionnaire questions in KS3, but this is not on the GCSE specification.

All students learn about choosing a random sample to avoid bias in GCSE Maths. They should understand the concept of random numbers and using these to generate a random sample, but they may not know how to generate random numbers on a calculator.

Only Higher level students learn stratified sampling and the capture-recapture method in GCSE Maths.

Approach

Estimating from samples

Use arrow diagrams to scale up

For example, a quadrat 0.5 m × 0.5 m is thrown in a field 5 times.

The number of dandelions in each quadrat is counted. The mean is 10 plants.

The field has area 2000 m^2.

Estimate the number of dandelions in the field.

		Area	Dandelions	
Sample		0.25 m^2	10	
	$\times 4$			$\times 4$
In 1 m^2		1 m^2	40	
	$\times 2000$			$\times 2000$
Field		2000 m^2	800 000	

Terminology

- **<u>Inconsistency</u>:** In maths students learn that data is either continuous, discrete (rather than discontinuous) or categorical. Data that is measured is continuous.
- **<u>Inconsistency</u>:** In maths students do not manipulate data (which has negative connotations) – they process it.
- Good questionnaire questions have tick boxes for answers to reduce answer options, make sure ranges for the tick boxes do not overlap, and all options are covered, are set in a clear time frame (e.g. how many magazines do you read each month, rather than just 'how many magazines do you read?'), are not be leading or biased. A survey should be anonymous in order to get truer answers.
- In a random sample, every member of a population has an equal chance of being selected.
- To select a random sample of size 10 you can:
  - o give every population member a number
  - o generate 10 random numbers and select the members with those numbers for the sample.
- For a small sample size you can put 'numbers into a hat' and draw them out at random. This is not practical for a large sample size.
- A sample that is too small can give biased results.
- For data that is grouped e.g. by age into strata, you can take a stratified sample, so the sample reflects the proportion of each age group in the population.
- You can use lines of best fit or follow the trend of a graph to estimate 'missing' data values. Estimating values that lie within the range of given values is called interpolation, though students may not learn this term. Estimating values that lie outside the range of given values is called extrapolation, though students may not learn this term. Interpolation is more likely to be accurate than extrapolation.
- Capture-recapture method for estimating the size of a population N: capture a sample of M items and mark them. Release them back into the population. Capture another sample of size n. Count the number of marked items, m. Assuming the proportion of marked items in your sample is the same as for the whole population $\frac{M}{N} = \frac{m}{n}$, so rearranging gives $N = \frac{Mn}{m}$.

17. Bias, error

For Bias, see section 16 Sampling and data.

For Error, see section 13 Significant figures, decimal places, accuracy.

Get in touch

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